

Happy Birthday AlgebraicDynamics.jl

Berkeley Seminar, Topos Institute

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Happy Birthday AlgebraicDynamics.jl

A Julia package for operad algebras of dynamical systems.

Resource Sharers

Resource Sharers - Compartmental Models

Step 1 Define the primitive systems.

```
= b s O ( Z i b Wh ] ] c b z Y Wh ] c b S X r i n b d d l a k ] W g
      ' ' ' G z ' = ' 1 ' i '
' ' ' ] b Z Y Wh ] c b ' S ' Z ' c G k = '
' ' ' O ] b Z Y Wh ] c b z S ] Z b ' z c y k W h ] c b S Z ' c k
Y b X
] b Z Y Wh ] c b S i a z & b h ] b i c i g F Y g c i o f W & G h q u & ' & f ' b Z Y Wh ] c b S X r i n b d d l a k ] W g
```

C i h O'

7 c b h] b i c i g F Y g c i f r W ' G R U & E Y k f j h \ ' & ' Y l d c g Y X ' d c f h g

```
= b s O ( Z i b Wh ] ] f c y b W c j Y f m S X r i n b d d l a k ] W g
      ' ' ' = z ' F ' 1 ' i '
' ' ' f ' Y W c j Y f m S i z d ' d k '
' ' ' O f Y W c j Y f m S Z Y W k j Y f m S Z ' c k
Y b X
f Y W c j Y f m S a ' d X ' b h ] b i c i g F Y g c i o f W & G h q u & ' & f ' f Y W c j Y f m S X r i n b d d l a k ] W g
```

C i h O (

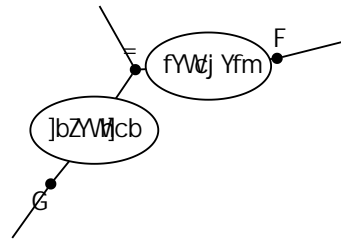
7 c b h] b i c i g F Y g c i f r W ' G R U & E Y k f j h \ ' & ' Y l d c g Y X ' d c f h g

Resource Sharers - Compartmental Models

Step 2 Define the interaction pattern.

```
= b s O ) Wc a d c g ] h ] c b S d U h i Y Y U b h j G z b = z F t V Y [ ] b  
] b Z Y W h f G z c t  
f Y W c j f Y = E i n t  
Y b X  
h c S [ f U d f W c ] a m c g ] h ] c b S d U h i h S Y f U b V Y i G U a Y i b W h ] c b S U V j Y U G ] U V Y
```

C i h O)



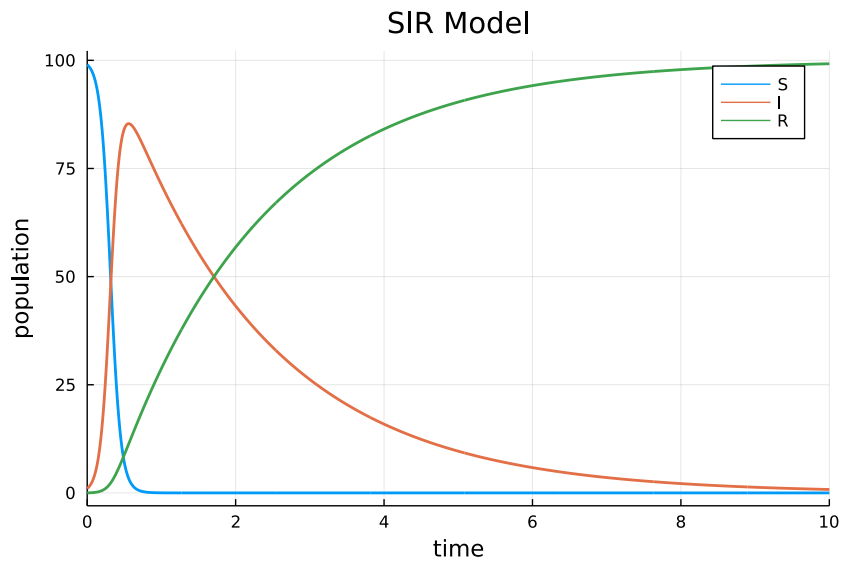
Resource Sharers - Compartmental Models

Step 3 Compose!

```
= b s O *G= F S a c' X' Y U d d f W a d c g ] h ] c b S d' U h h Y f b
      '8] W h ] b Z Y W h ] 2] b b Z Y W h ] c b S' a d' X' W c j' Y 2 f m Y W c j Y f m S a d' X' Y`
```

```
= b s O , i $ 1' O - "z$%" z$ "$ G
      d U f U' a @ J Y W h i c' f' $ " %) ' 1' $ " j' ' 1' $ " $ %
h g d U i b f i $ " z' % $ E'
      d f c' V' C 8 9 D f c' V G = Y F a S a c' z X i Y $' h g d U' U f U t a g
g c' 1' g c' ' f i d Y f c' z V H g ] f i ] t
      d' d f i g c' z' G = F S a c' z X' Y K & z' h ] h 1' Y G = F' A c' z X i Y' U V i Y' h ] a z Y m' U V i Y' d c d i' U h ] c b`
```

C i h O ,

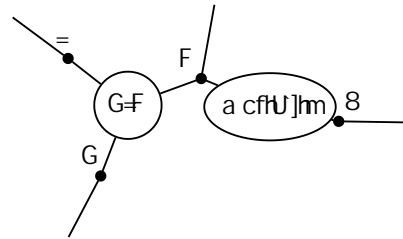


Resource Sharers - Compartmental Models

Hierarchical composition!

```
= b s O - Wc a d c g ] h ] c b S d 1 U 4 h n Y f b b & ] G z b = z ' F z ' 8 E ' V Y [ ] b
      ' ' ' G = F G z ' = z ' F E '
      ' ' ' a c f h U ' f i F z h 8 h '
Y b X
h c S [ f U d f W c ] a m d c g ] h ] c b S d ' U h t h ' S f b d & ' 1 Y ' ' t g U a ' Y i b W h ] c b S ' 1 U . V j Y U f g ] U E V ' Y
```

C i h O -



```
= b s O % a c f h U ' ] h m S X f i n b d z h a t ] 1 W G d " ' i ' i O % Q z d " ' i ' i O % Q Q
      a c f h U ' ] h m S l a z & X Y h ] b i c i g F Y g c i o f W e G t q U E f ' 8 f ' a c f h U ' ] h m S X z m b U a ] W g
```

C i h O %

7 c b h] b i c i g F Y g c i f R W ' G R U & E Y f k f j h \ ' & ' Y I d c g Y X ' d c f h g

```
= b s O % G = F 8 S a c 1 X d U d f W c a d c g ] h ] c b S d ' U h h Y f b &
      ' ' ' 8 ' ] W h G = F 2 G = F S a c z X . Y a c f h U ' ] 2 h a r c f h U ' ] h m S b i c X Y '
```

C i h O %

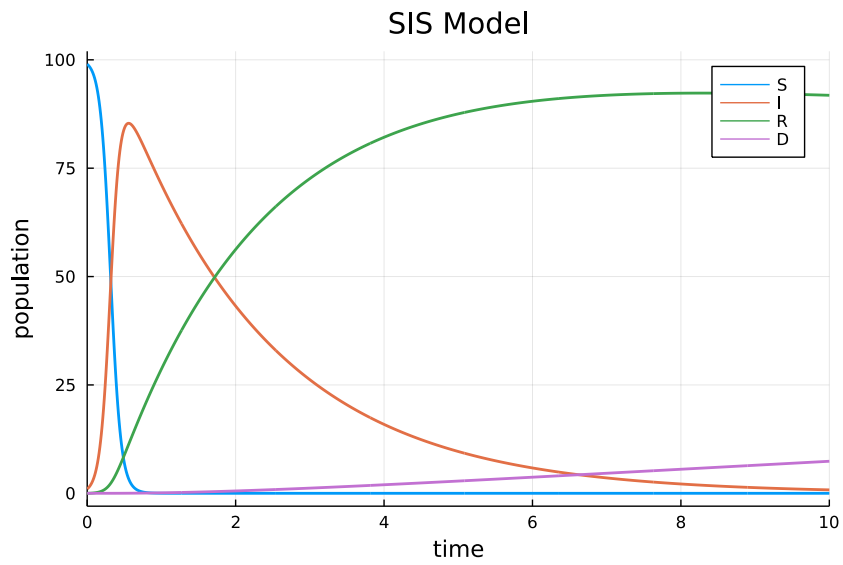
7 c b h] b i c i g F Y g c i f R W Y G R U & E Y f k f j h \ ' (' Y I d c g Y X ' d c f h g

Resource Sharers - Compartmental Models

Hierarchical composition!

```
= b s O % i $1'0- - "z$%" 本 $" 本 $" 本  
d f c'V' C 8 9 D f c W G =Yfa8 S a e'X Y' h g d U f Uta g  
g c "1'g c ' jldyf c'zVH g ] fi k t  
d ` d'lg c z'G = F 8 S a e'X W &z' h ] h 1'YG = G ' A c zXIY " U V IY 'h ] azY m' U V IY 'd c d i ' U k ] c b "
```

C i h O %



Resource Sharers - Springs

An anchored spring

```
= b s O % Z i b Wh ] U b W \ c f Y X S g d f ] b [ f i S t X m b U a ] W g
    . . . . . f l i z d z h t ! ! 2 V Y [ ] ' b
. . . . . a ' U g g d c z j Y ' 1 ' i '
. . . . . O $ ' z ' j Y z ' ! _ l d c a U g g
. . . . . Y b X
. . . . . Y b X
. . . . .
U b W \ c f Y X S g d f ] b [ f i S t X m b U a ] W g
```

```
C i h O %
U b W \ c f Y X S g d f ] b [ ' f l [ Y b Y f ] W ' Z i b Wh ] c b ' k ] h \ ' % ' a Y h \ c X t
```

```
= b s O % _ ' ( " $
g d f ] b [ U b W \ c f Y X S g d f ] b [
```

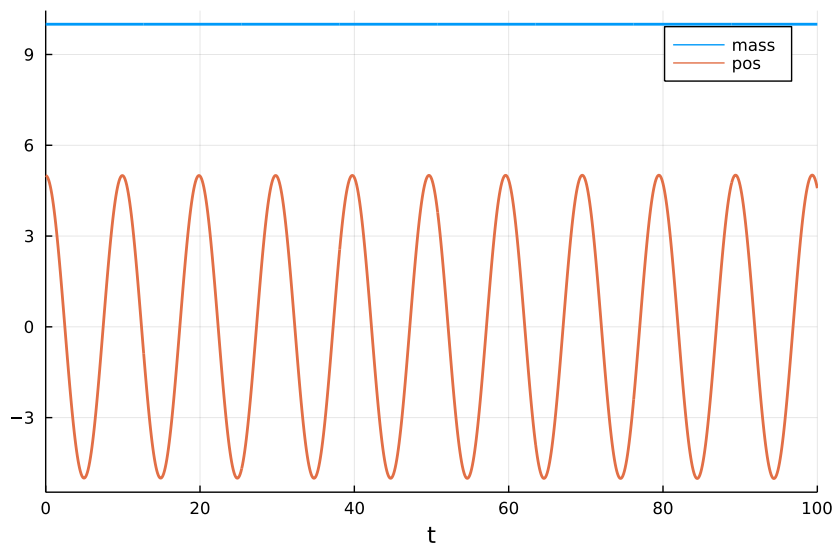
```
C i h O %
7 c b h ] b i c i g F Y g c i f R W Y ' G ' R U f l Y f k f ] h \ ' & ' Y l d c g Y X ' d c f h g
```

Resource Sharers - Springs

An anchored spring

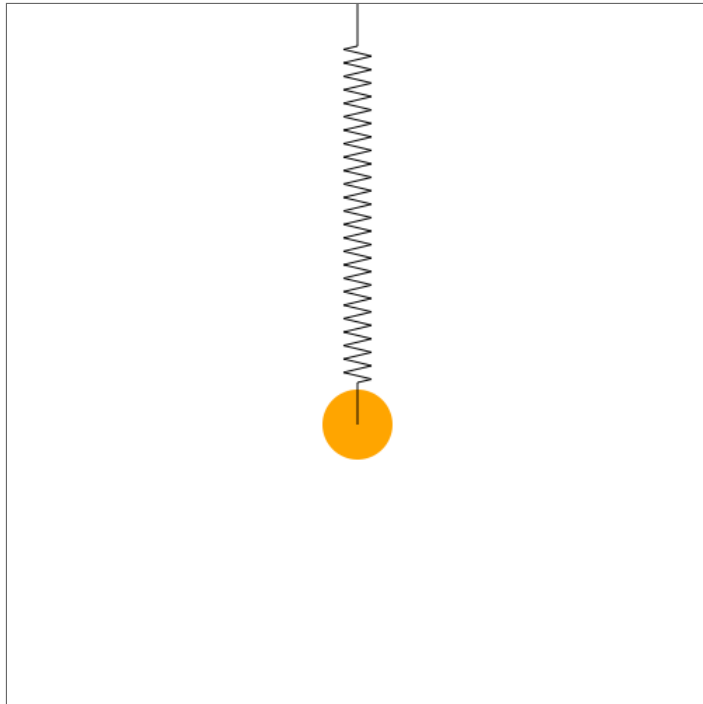
```
= b s O % a U g g' % $ "/$d $1' ) " , $  
i $ 1' Ca U ggd $ '$ 'Q'  
d U f U 'a g@J Y Wff[cif " t / h g d U1b fl$z' % $ $ E' $  
g d f ] b [ S'd fCc8V9 D f c Wg dYfa] zb i[ $' h g d b'ld U f Uta g  
g d f ] b [ S'lg@c'` jlgYd f ] b [ Szd Hf @Vf] : Zg U j Y1$ H E%  
d ` cf]g d f ] b [ S'gg d'f ] zb` [ k 1' &E'
```

C i h O %



Resource Sharers - Springs

An anchored spring



Resource Sharers - Springs

An anchored spring... with gravity!

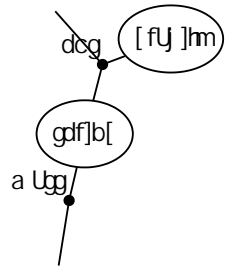
```
= b s O % [ f U j ] h m S a 1 c 7 X c y b h ] b i c i g F Y g c i o f W & Q h q 0 % & f f l i z d z h t ! ! 2 O O & Q z d " [ O z % % & .
```

C i h O %

```
7 c b h ] b i c i g F Y g c i f R W G R U & E Y k q h \ . % . Y I d c g Y X . d c f h
```

```
= b s O % W c a d c g ] h ] c b S d l a f i P Y f U t h f i a c U b g z g d c t g V Y [ ] b  
g d f ] f i a U g z g d c t g  
[ f U j ] f i d c r i g  
Y b X  
h c S [ f U d f W c j a m l c g ] h ] c b S d V U c h I h S f U b V I Y . t g U a . Y i b W h ] c b S . 1 U . V j U t g ] U & V . Y
```

C i h O %

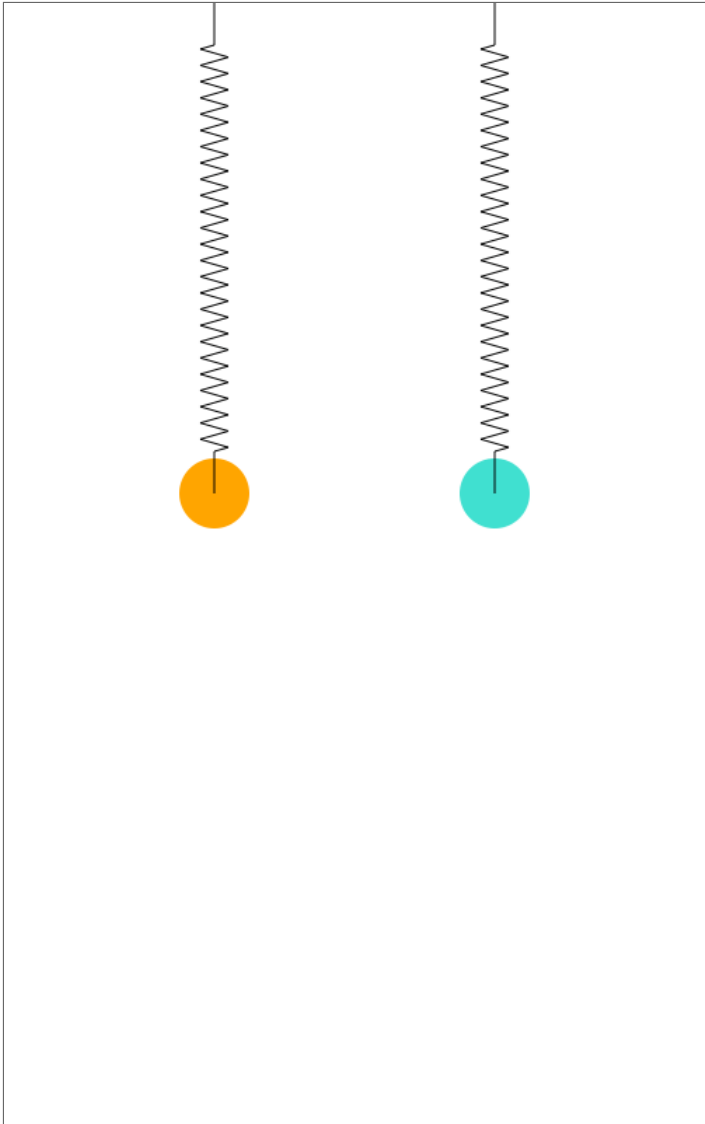


Resource Sharers - Springs

An anchored spring... with gravity!

= b s O & g d f] b [S c b S i Y d l f d f W m a d c g] h] c b S d 8 J h V i h Y g d b f] b 2 g d f] z b . [[f U j] 1 h 2 h f U j] h m S l a t c X Y `

= b s O & i \$ 1 ' C a U g d \$ ' \$ ' z ' \$ ' 0 ' g d f] b [& S t C 8 9 D f c W g d Y f a] b [S c b S Y S f h g d b ' l U f U l a g g d f] b [& S g g c ' ' f l g / d f] b [& S d H f g c j W i } z g U j Y 1 \$ H 0 % /



Resource Sharers - Springs

Springs in parallel

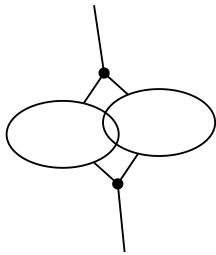
```
= b s O & _ % ! % $ " / $ _ & ! ' & $ " ' $  
g d f ] b [ ' % b W \ c f Y X S g_d% ] b [  
g d f ] b [ ' % b W \ c f Y X S g_d% ] b [
```

C i h O &

7 c b h] b i c i g F Y g c i f R W Y G R U f l e Y f k f j h \ ' & ' Y l d c g Y X ' d c f h g

```
= b s O & W c a d c g ] h ] c b S d ' % f i Y Y f U b h j l a c l b g g d c l g ' V Y [ ] b  
g d f ] b [ ' % b W \ c f Y X S g_d% ] b [  
g d f ] b [ ' % b W \ c f Y X S g_d% ] b [  
Y b X  
.  
h c S [ f U d f W c j a m l c g ] h ] c b S d V U c h l h S y f l b V i Y : l g U a ' Y i b W h ] c b S ' 1 U . V j U f g ] l t V ' Y
```

C i h O &



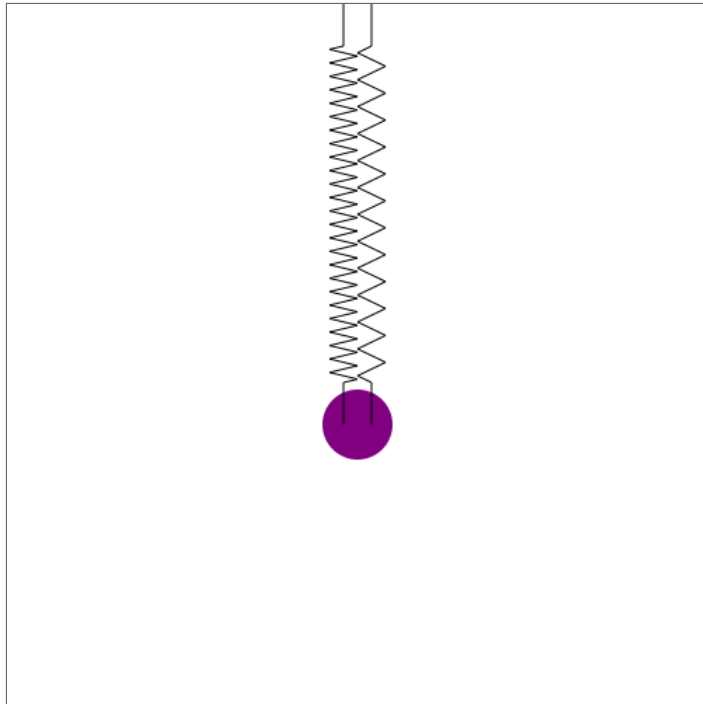
```
= b s O & d U f U ' ' Y ' S g ' d f c ] U b d [ d g W m a d c g ] h ] c b S d Q g h f Y f z t g % f ] b [ E &
```

C i h O &

7 c b h] b i c i g F Y g c i f R W Y G R U f l e Y f k f j h \ ' & ' Y l d c g Y X ' d c f h g

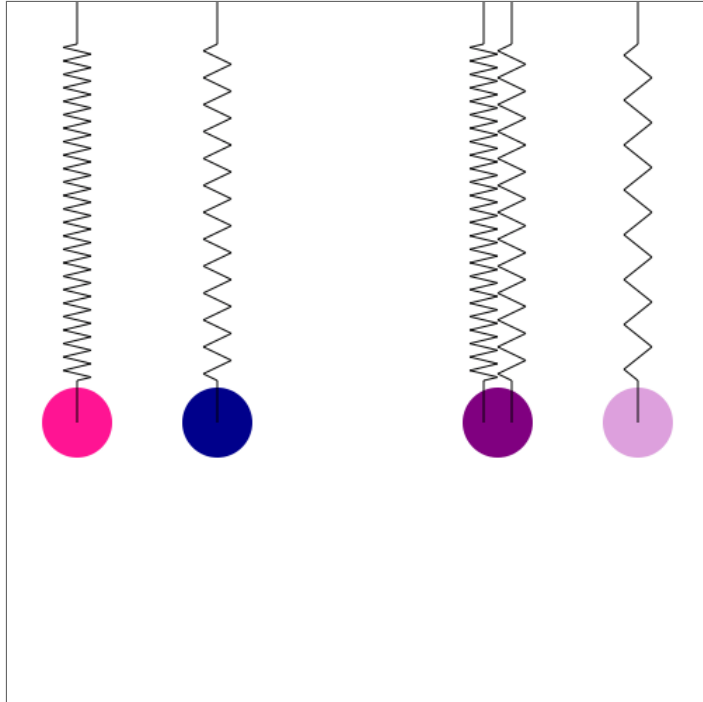
Resource Sharers - Springs

Springs in parallel



Resource Sharers - Springs

Springs in parallel



Resource Sharers - Springs

Springs in series

```
= b s O & Z i b Wh ]ZcfbY Sg d f ] b [ S _l_m b U a ] Wg  
    . . . . . fli z dz h t : ! 2 V Y [ ] ' b  
    . . . . . h c d S a J g d S d chg: d S jz W c h S a J y g h S d cVg: h S j' Y i '  
    . . . . . OS z' h c d S jz Y l f h c d S d c' y c h S d chg: d S a J g z g V c h S jz Y l f V c h S d lchg: d S d cVg: h S a O g g  
    . Y b X  
    . Y b X  
    .  
Z f Y Y S g d f ] b [ 7 c b h ] b i c i g F Y g c i o f W x O h q u f ' Y f' Z f Y Y S g d f ] b [ S _l_m z O % & z W g O l
```

C i h O &

Z f Y Y S g d f] b [' f l [Y b Y f] W' Z i b Wh] c b ' k] h \ ' % ' a Y h \ c X l

Resource Sharers - Springs

Springs in series

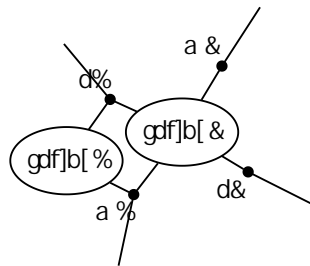
```
= b s O & _ %1 % $ "/ $ _ & 1 ' & $ " ' $  
g d f ] b [ % b W \ c f Y X S g_d % ] b [  
g d f ] b [ % f Y Y S g d f _ ] & b [
```

C i h O &

7 c b h] b i c i g F Y g c i f R W Y ' G R U f l e Y f k f j h \ ' (' Y l d c g Y X ' d c f h g

```
= b s O & g Y f ] Y g S W c a d c g ] h ] d b 4 S f d U H U h Y f a c b a & ' d % ' d & ' V Y [ ] b  
g d f ] b i d % d %  
g d f ] b i d % d % ' a & ' d & '  
Y b X  
h c S [ f U d f j Y f r j Y g S W c a d c g ] h ] z c b S d S h h W Y f b g U a ' Y i b W h ] c b S ' U V j Y U g ] U V ' Y
```

C i h O &



```
= b s O & g Y f ] Y g S g d f i ] c b [ d g f i g r f ] Y g S W c a d c g ] h ] z c b S d U h h Y f b ] U V &
```

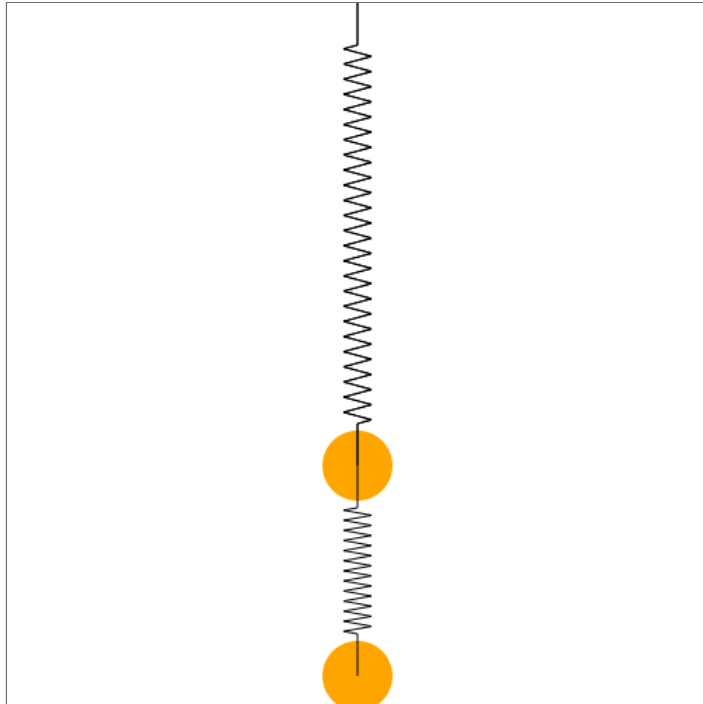
C i h O &

7 c b h] b i c i g F Y g c i f R W Y ' G R U f l e Y f k f j h \ ' (' Y l d c g Y X ' d c f h g

Resource Sharers - Springs

Springs in series

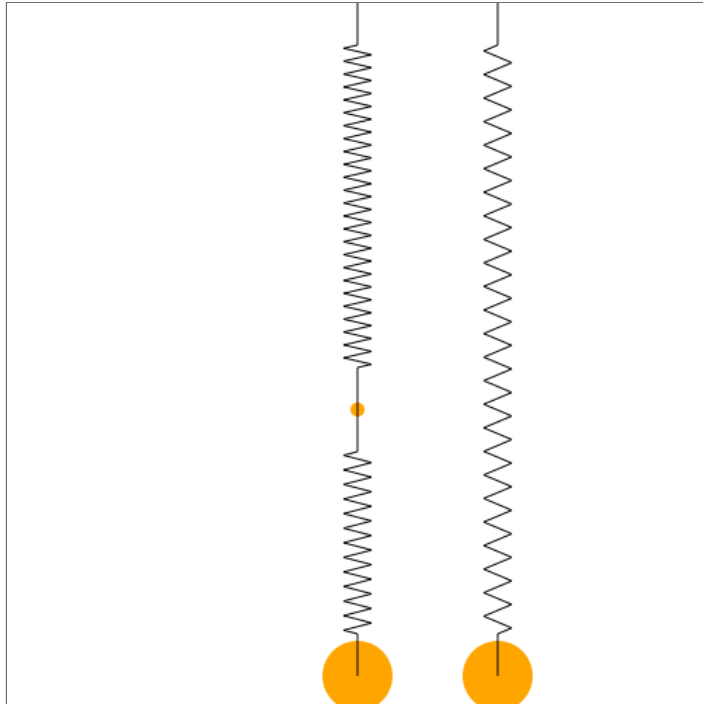
= b s O & a %1'%'\$"/\$d %1' (" \$
a &1'%'\$"/\$d &1', " \$
.
d f c`V`C 8 9 D f c Vg Wfa] Y g S g d f Q a b% [d g% \$ "z`\$ "z`a &`d & \$ "z`Q z h g d U`u U f U l a g
g c`1`g c` j l d y f c z V H g] f i } z g U j Y 1 \$ H B % /



Resource Sharers - Springs

Springs in series

= b s O ' a %1 '\$ " \$/)d %1' (" '\$
a &1' %\$ "/\$d &1', " '\$
.
d f c' V C 8 9 D f c Vg Yfa] Y g S g d f Q a b [d g % '\$ ' z '\$ ' z ' a & ' d & '\$ ' z ' Q z h g d U ' u U f U l a g
g c ^ 1 ' g c ^ j l d y f c z V H g] f i } z g U j Y 1 \$ H B % /



Machines

Machines - Fibonacci

```

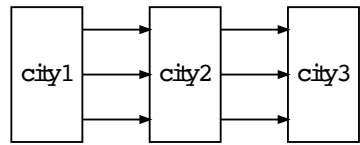
bÁI AG é Á Á Á Á Á Á Á   Á
  é Á Z F E Á e Á Á Á   Á
Á
  Á
] : [ÁBÁÖ ä • & ; ^ c ^ Ú Ç - [ää] [^]{æÁ * & Ç É Á P D É } [ c @ ð Á *
• [Á Á Á • [ | Ç ] ^ ! [É Á Ø ~ } & c ā [Ç D T Á e ]
Á
  Á
{ æ Ç • [ D Á X c Á c Á
Á Á ^ æ á Ç ~ æ à [ } æ Á c Ø É á Á
Y b Á

```

GF È ^ | ^ { ^ } c Á X ^ & c Á ! , Q } c í l ð K
Á Á Á Á €
Á Á Á Á F
Á Á Á Á F
Á Á Á Á G
Á Á Á Á H
Á Á Á Á í
Á Á Á Á ì
Á Á Á Á H
Á Á Á Á F
Á Á Á Á I
Á Á Á Á í
Á Á Á Á J
Á Á F Á I
Á Á G Á H
Á Á H Á Ì
Á Á í Á €
Á Á J Á Ì
Á F í Á Ì
Á G í Á I
Á I F Á F
Á í í í í

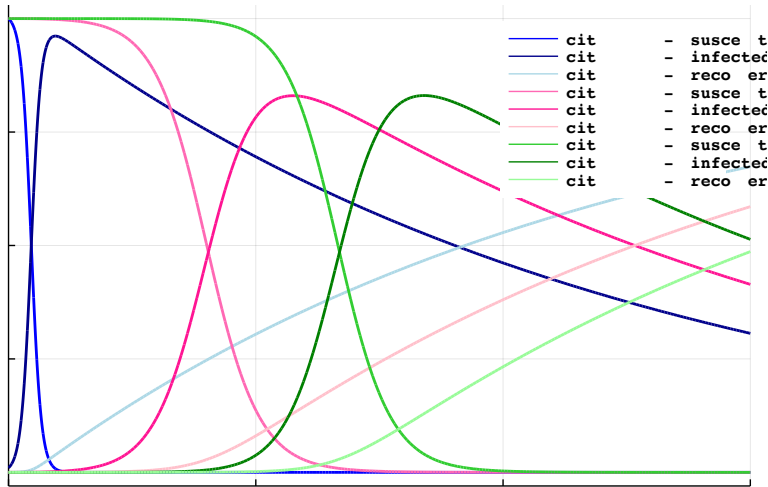
Machines - Multi-city SIR

} & a c ă 1 4 HLÁ! [æ Á Á Z F Á 1 2 Á G É Á G Á 1 2 Á H á Á
 & a c ^ ^] æ á c Á ^ Á ^ & a c ^ ^] Ç æ & a c É Á ! • [æ B Á
 c [^ * ! æ] Ç æ & a c ^ ^] æ É Á [Á ! á) ^ } c æ á á [Á V [Ú Á * @ c

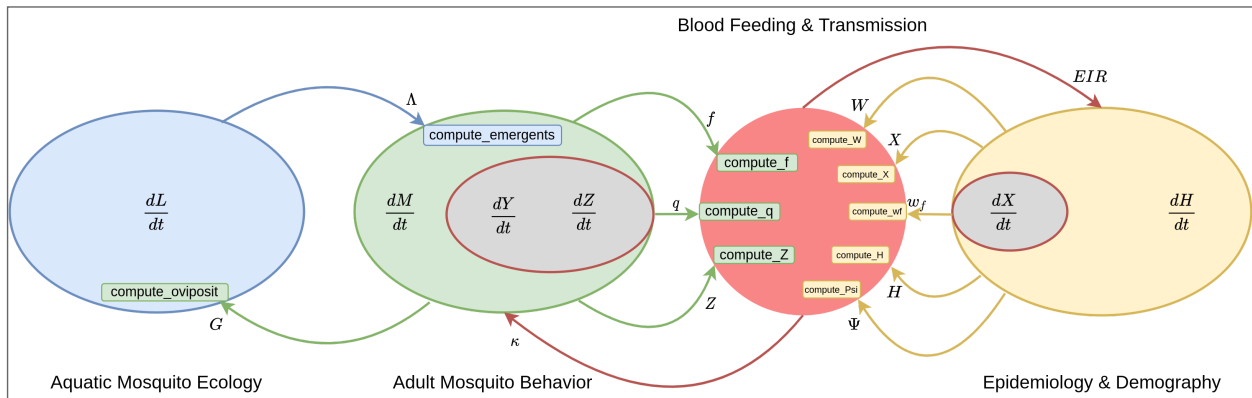


{ ^ | c á ' & a c Á Á [è Q Û Ç æ a c ^ ^] æ É Á ! V Q K & a c Á É & a c É Á K & a c Á G & a c É Á K & a c Á É & a c D B Á

á á •] Ç æ [D Á



Instantaneous Machines



Wu S, Smith D (2022). *MicroMoB: Discrete Time Simulation of Mosquito-Borne Pathogen Transmission*. <https://dd-harp.github.io/MicroMoB/>, <https://github.com/dd-harp/MicroMoB>.

Instantaneous Machines

```
plus machine = DiscreteMachine Int ->
```

is eta a ine

```
open in plus machine
```

element eta ai nt nt

```
eval name plus machine -
```

nt

```
eval out plus machine -
```

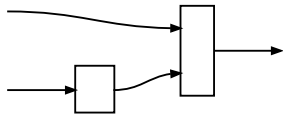
element eta nt

Instantaneous Machines

ela ma ine `DiscreteMachine Int` u p t -> u p t -> u

is ete a ine

to ap iz i in patte n o ientation = e t o i t



o set plus = oappl i in patte n `Dict i =>` ela ma ine: => plus ma ine

is ete a ine

epen en pai os set plus

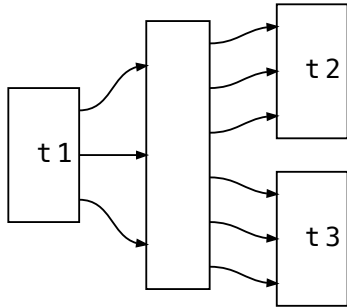
element e to ai nt nt

Instantaneous Machines

```
s =  
s = -  
  
u = s :  
  
for t in :  
    print ea outo set plus u st st n  
    u = e al nami s set plus u st st  
end
```

Instantaneous Machines - Multi-City SIR

to ap iz it patte n o ientation = e t o i t



o = ContinuousMachine load64 -> at

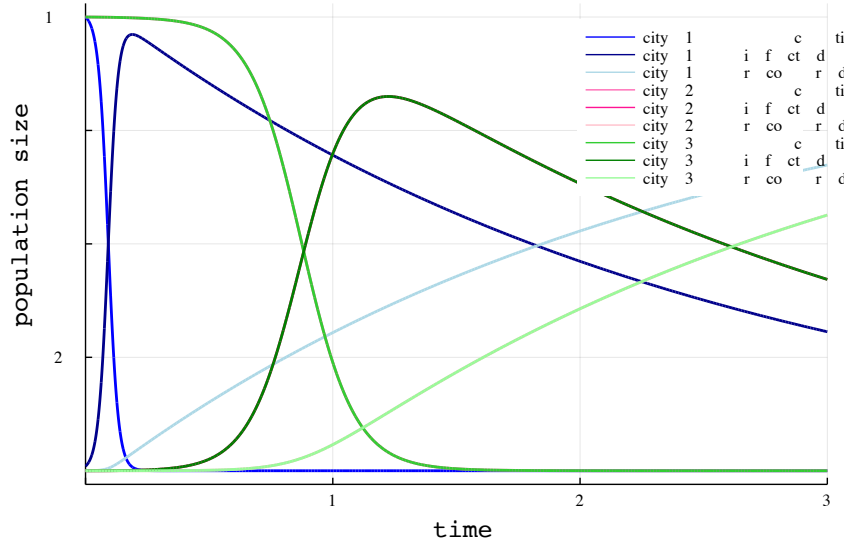
ontinuous a ine

multi it = oappl it patte n Dict it => it it => it it => it o => o

ontinuous a ine

Instantaneous Machines - Multi-City SIR

ispla plt



Deeply Interacting Learning Systems (DILS)

```
= b s 0) relu - a nt u u
```

```
= b s 0 *function unacted neurom in ut nt Float .
      Float
      for in n in ut n ut
      n in ut ut ut
      u - dot u cat t t t nt
      u - u end- ut ut tu n
      u - u - cat n u tu n
end
acted neuromin ut nt
e uential co unacted neuromin ut relu
```

C i h 0 *

```
acted neuron eneric function it et od
```

Deeply Interacting Learning Systems (DILS)

```

= b s 0 %      fun      a      un t n t      n
      fun      fun

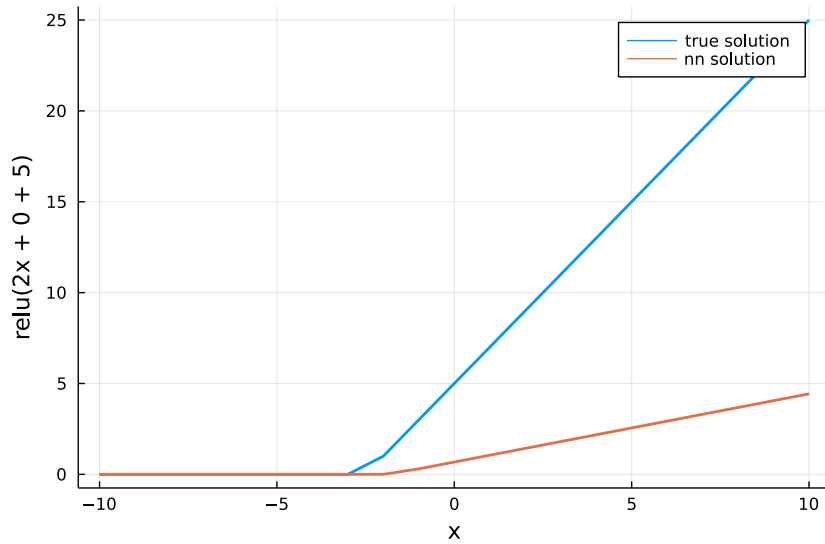
ei t rand
ia rand
nn Sin le euron ei t ia acti ated neuron
initiali e nn
    
```

```

= b s 0 %      -      fun trin relu + trin +
true olution a - fun
nn olution a - nn

lot true olution l la el true olution la el la el fun trin
lot nn olution l la el nn olution
    
```

Ci h 0 %

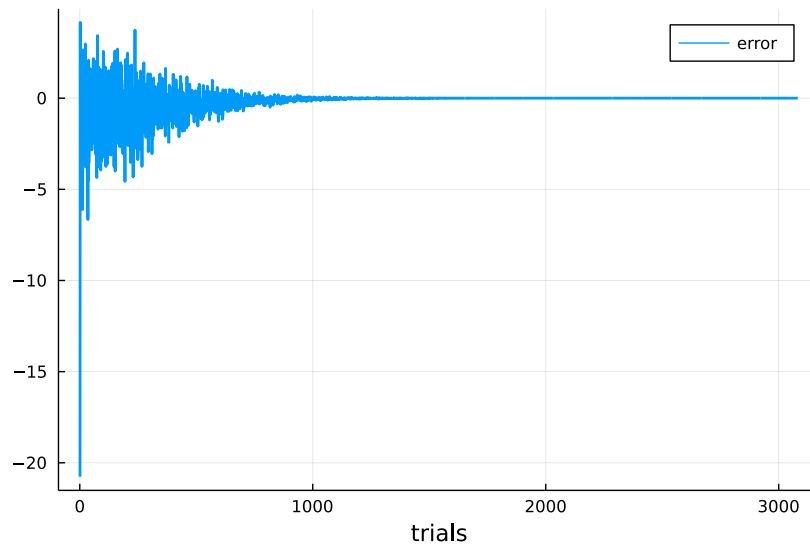


Deeply Interacting Learning Systems (DILS)

```
= b s 0 % data trainin data fun
err train nn data

non ero err collect filter - err
lot len t non ero err non ero err l la el error la el trial
```

C i h 0 %

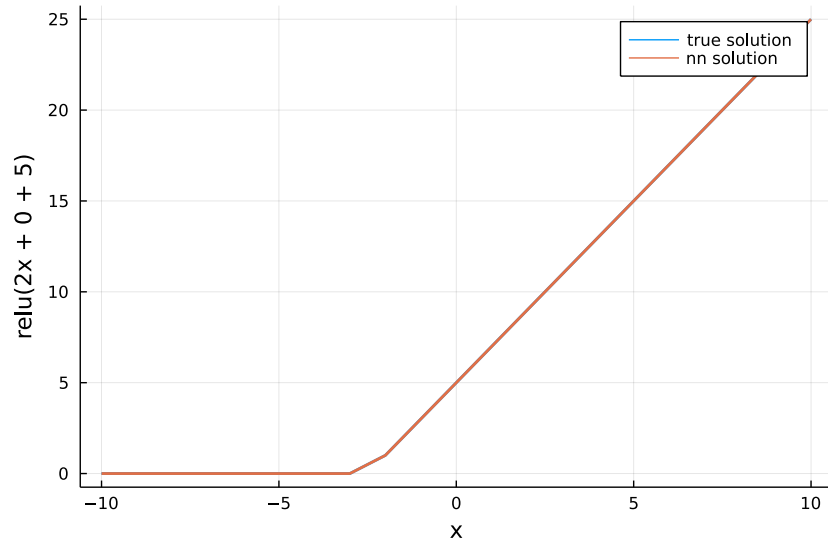


Deeply Interacting Learning Systems (DILS)

```

= b s 0 % - fun trin relu + trin +
true olution a - fun
nn olution a - nn
lot true olution l la el true olution la el la el fun trin
lot nn olution l la el nn olution
    
```

C i h 0 %



The core AlgebraicJulia team



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Topos Institute



Andrew Baas
GTRI



Owen Lynch
Utrecht University



Kris Brown
University of Florida



Micah Halter
Balena

References

AlgebraicDynamics

- Libkind, Sophie, Andrew Baas, Evan Patterson, and James Fairbanks. 2021. "Operadic Modeling of Dynamical Systems: Mathematics and Computation." <http://arxiv.org/abs/2105.12282>.

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