Co-Design of Complex Systems: From Autonomy to Future Mobility Systems

Topos Institute Colloquium

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Designing today's engineering systems could have positive societal impact, but is complex

- **Autonomous systems** as a proxy for **complex systems**, which might have **positive societal impact**

Autonomy for safer and efficient mobility (Motional)

Autonomous robots for space exploration (Pavone et al.)

Roboflies to monitor environments (Fuller et al.)

UAVs for search and rescue tasks (Scaramuzza et al.)

Need new tools to model and solve complex systems design optimization problems

- **Societal impact** of new technologies depends on their **joint design** with **existing systems**

Intermodal mobility networks (NASA UAM)

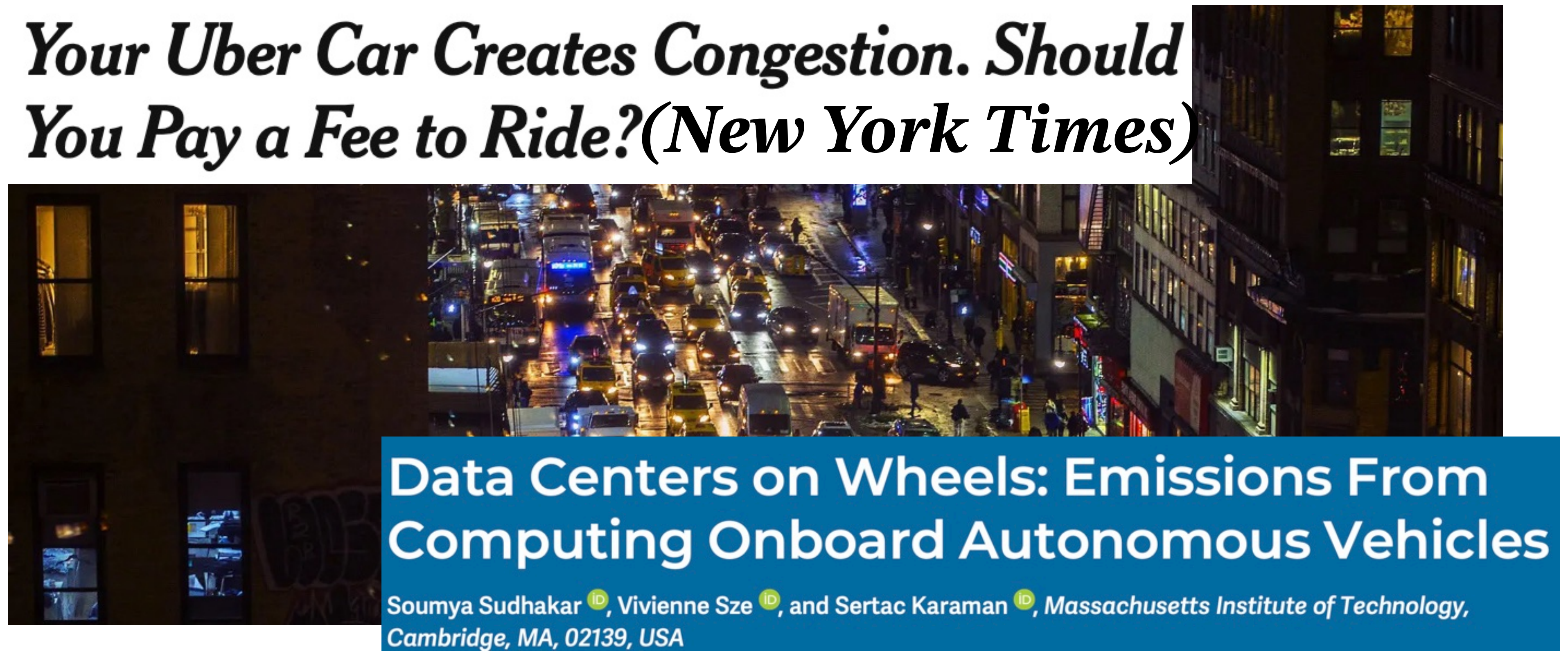
Networks of tankers (Signal Ocean)

*Example - Autonomy: **Heaven** or **hell**?*

***30%** of the cars would be enough*

First**- and **last-mile** mobility could make **public transit** more **convenient** and **attractive

*More **affordable**, **sustainable***



Single components are slowly well understood, but we still lack a (*formal* and *practical*) theory for the **task-driven co-design** of **complex systems**

Agenda

- **Motivation**
  - *New challenges of engineering design*
  - *Motivation from autonomy and mobility*
  - *Desiderata for co-design*
- **Monotone Co-Design**
  - *Modeling design problems*
  - *Examples across domains*
  - *Design queries and optimization*
  - *From autonomy to mobility systems*
- **Strategic interactions**
  - *Game theory to deal with strategic interactions*
- **Outlook on future research**

Website containing all papers and more pointers:

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Driven by **societal challenges**, I develop **efficient computational tools** to **automate** the **formulation** and **solution** of **large**, **complex system design problems**

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The vision of automated co-design

minimize
(resources usage)
subject to
(functionality constraints)

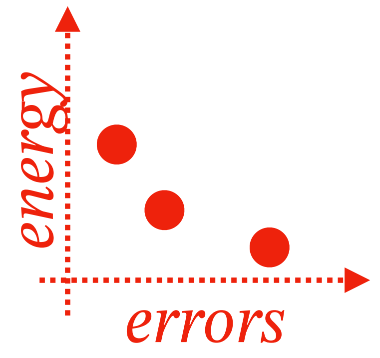
task specification →
multi-domain knowledge →
design options →

"automated designer"

→ **optimal design(s)**

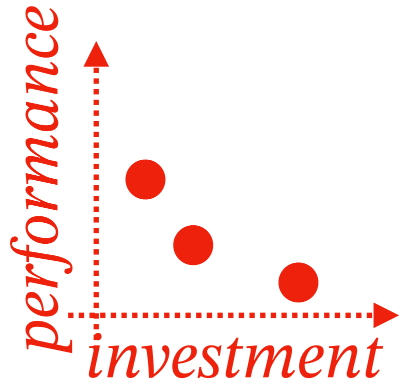
Autonomy co-design

task →
robot autonomy, physics →
components, algorithms →



Mobility co-design

demand →
networks, operations, infrastructure →
mobility services, policies →



Autonomy as the frontier of complexity for the co-design of complex systems

A fleet of autonomous vehicles

=

actuation
hardware
sensing
computation
energetics
software
localization
control
perception
communication
behavior
planning
interaction
mapping
coordination
invasivity
learning
liability
regulations
infrastructure

OMG!

So many **components** (hardware, software, ...), and **choices** to make!

Nobody understands the **whole** thing!

We forget why we made **choices**, and we are afraid to make **changes** (high failure cost).

We need **faster** design cycles, **nimbler** execution.

anthropomorphization of 21st century engineering malaise

"My dear, it's simple: you lack a theory of **co-design**!"

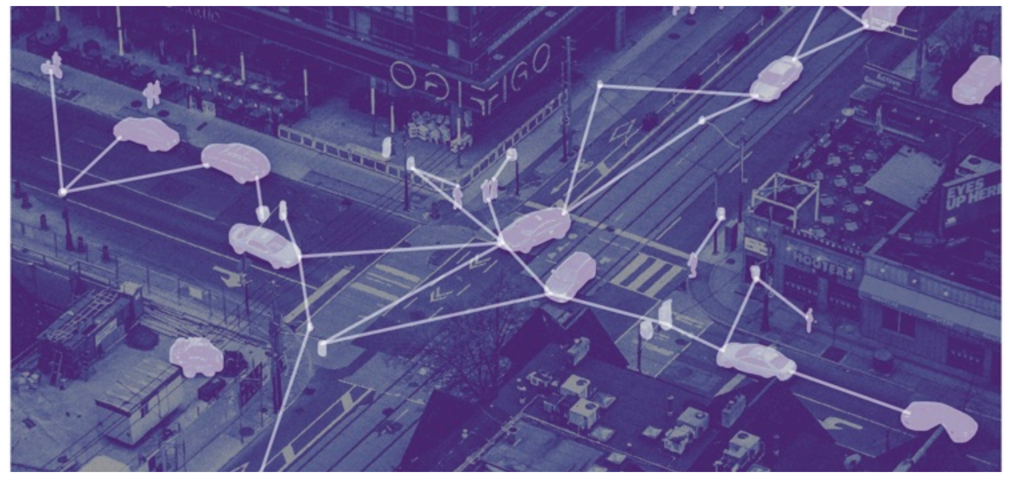
Formal
Quantitative
Intellectually tractable

Your system is just a component in another person's system

Infrastructure level

Optimal infrastructure choices

Service level



Optimal deployment

Platform level

Choice of components

Subsystem level

Single component design

Complex systems typically feature multi-stakeholders interactions

Mobility providers
(Jamey Tesler,
CEO MassDOT)

Liability
(Christian Mumenthaler,
CEO SwissRe)

Policy makers
(Michelle Wu,
Mayor Boston)

Academia
Tech Developers
(Sally Kornbluth,
MIT president)

Industry
(Laura Major,
CTO Motional)

Challenges for automated co-design of complex systems

Complexity when designing complex systems

Large systems
- Many components, scales
- Heterogeneous natures
- Multiple objectives

Strategic interactions
- Many agents
- Heterogeneous interactions
- Conflicts/collaborations

A fleet of autonomous vehicles

=

software hardware behavior coordination
actuation sensing localization planning invasivity
control interaction learning
computation perception mapping regulations
energetics communication infrastructure

Desiderata for the automation of complex systems co-design

- **Formal, domain-independent**
- **Computationally tractable**
  - Need to compute solutions efficiently
- **Compositional, hierarchical**
  - My system is a component of somebody else's system
- **Collaborative**
  - Pooling knowledge from experts across fields.
- **Intellectually tractable**
  - Not exclusively accessible to system architects
- **Continuous**
  - Design is not static: it should be reactive to changes in goals and contexts

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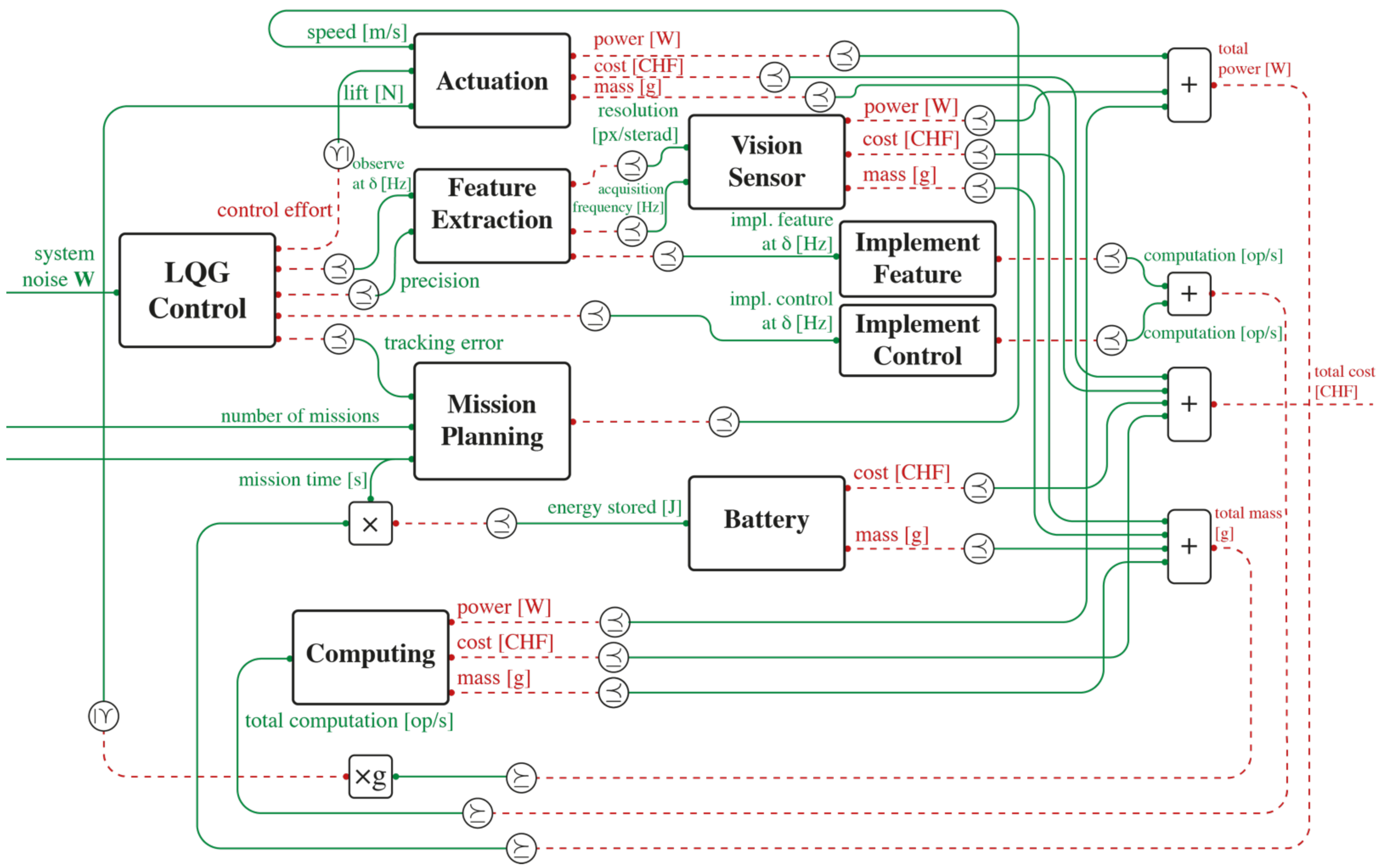
A new approach to multi-disciplinary engineering "co"-design

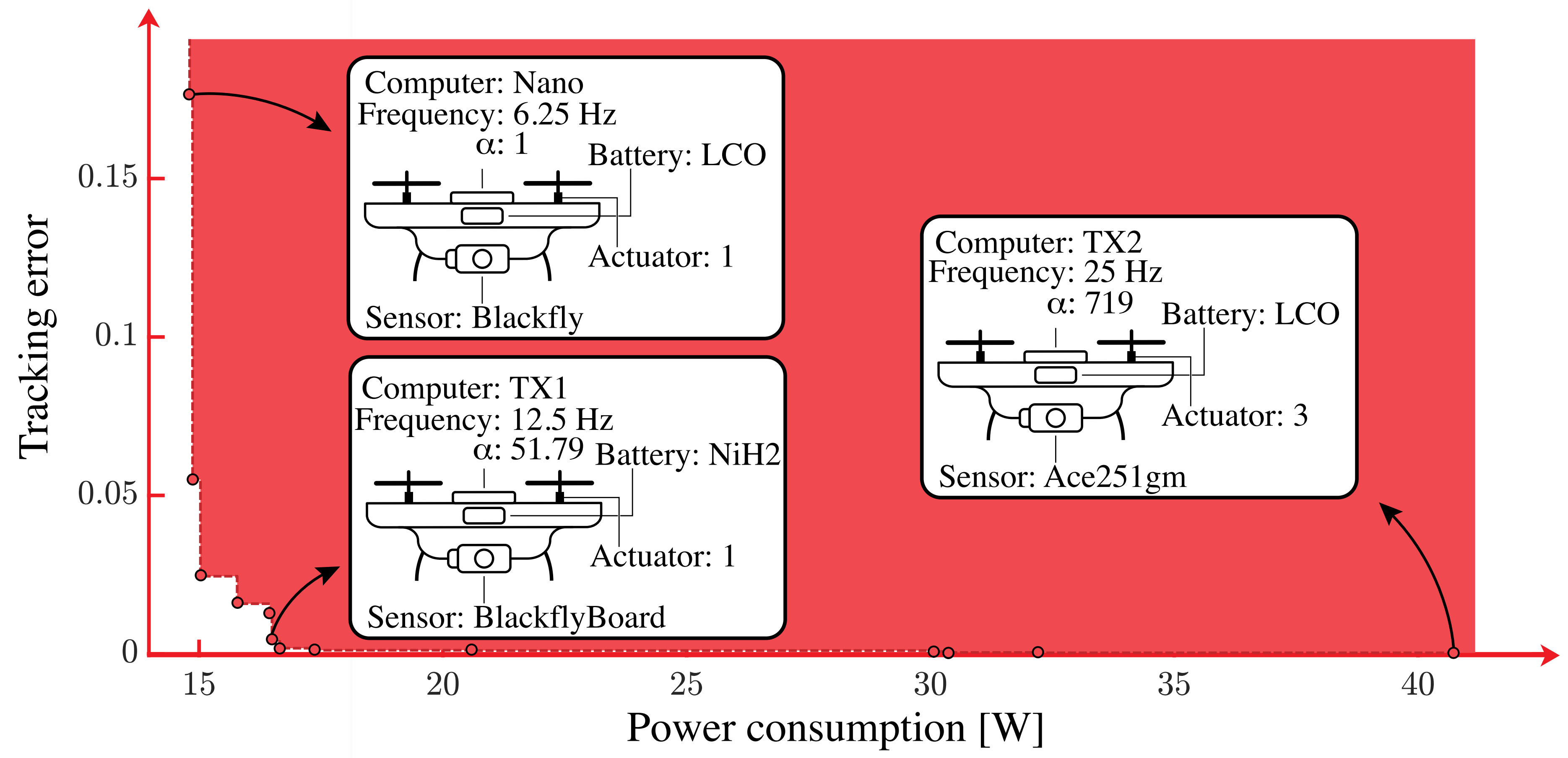
- A new approach to **collaborative**, **computational**, **compositional**, **continuous** design designed to work **across fields** and **across scales**.
- Leverages **domain theory**, **applied category theory**, and **optimization**
- Roadmap:
  - Defining **"design problems"** for **components.**
  - Modeling **co-design constraints** in a complex **system**.
  - **Efficient** solution to design queries.

"Co-design diagram"



optimization for a task

Pareto front of optimal designs



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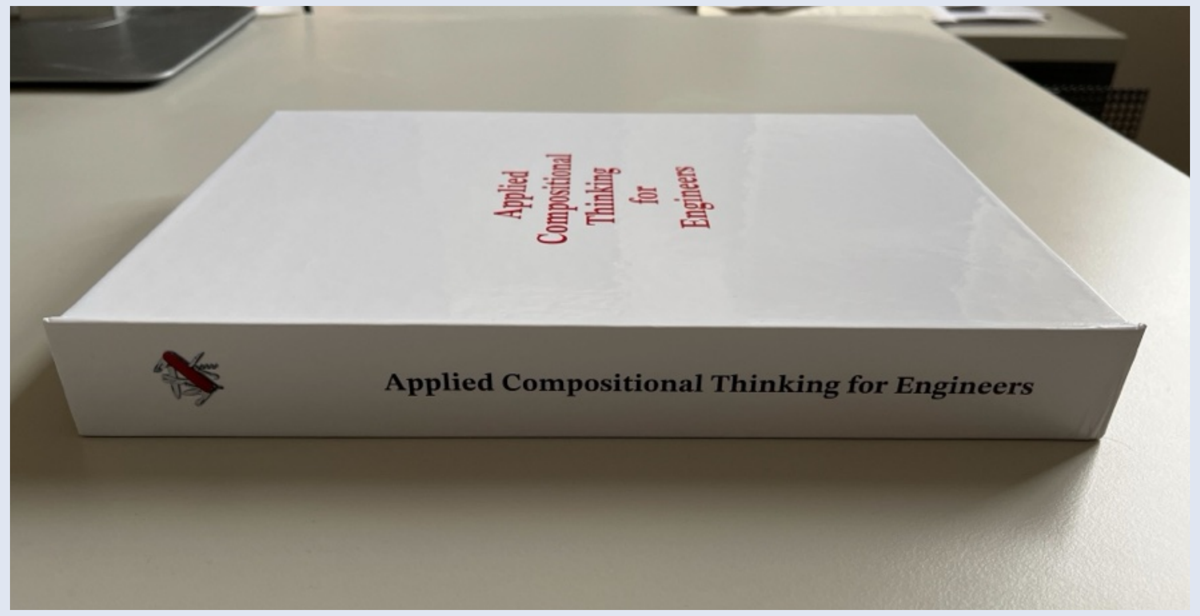

Access the book at:
https://bit.ly/3qQNrdR

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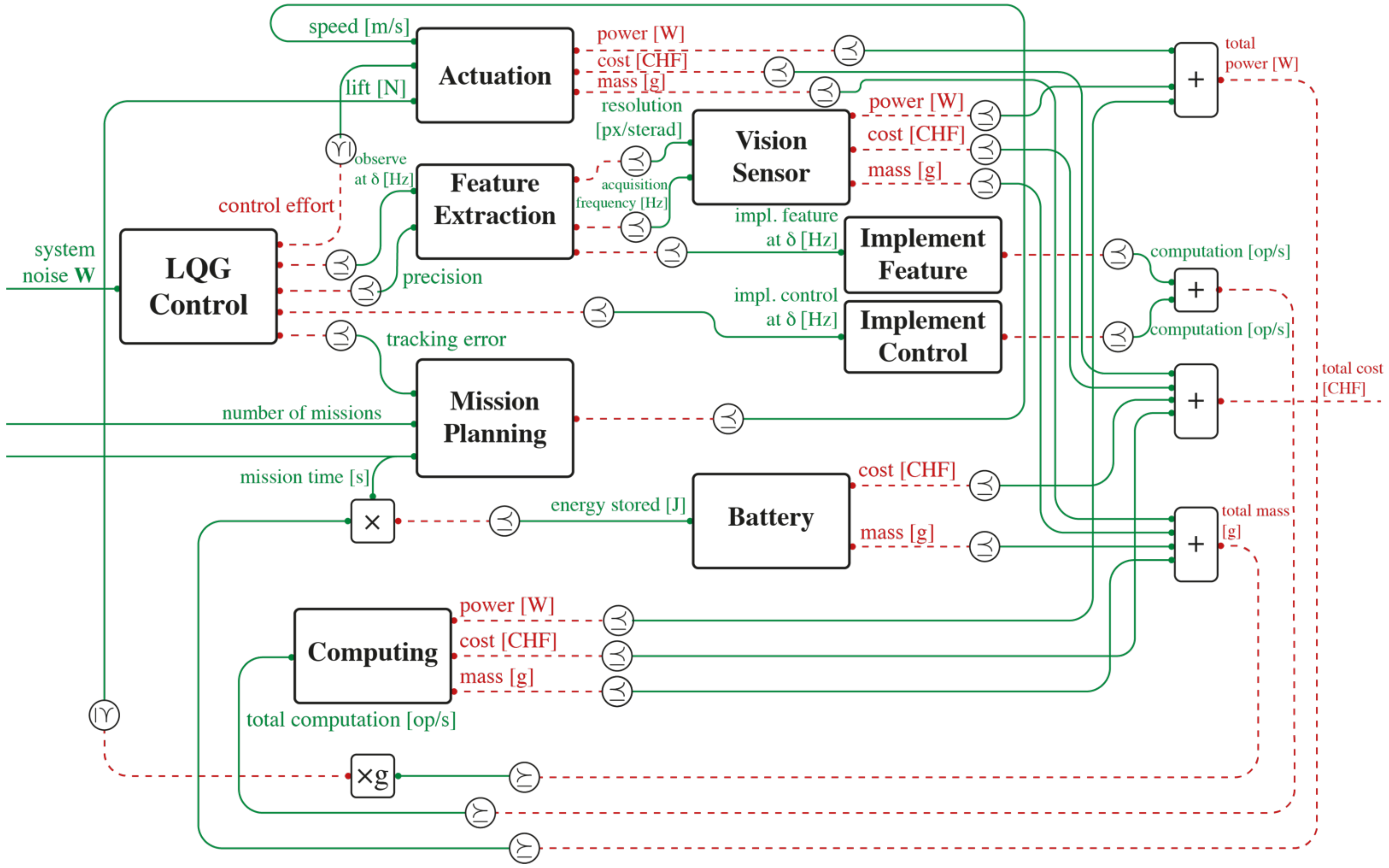

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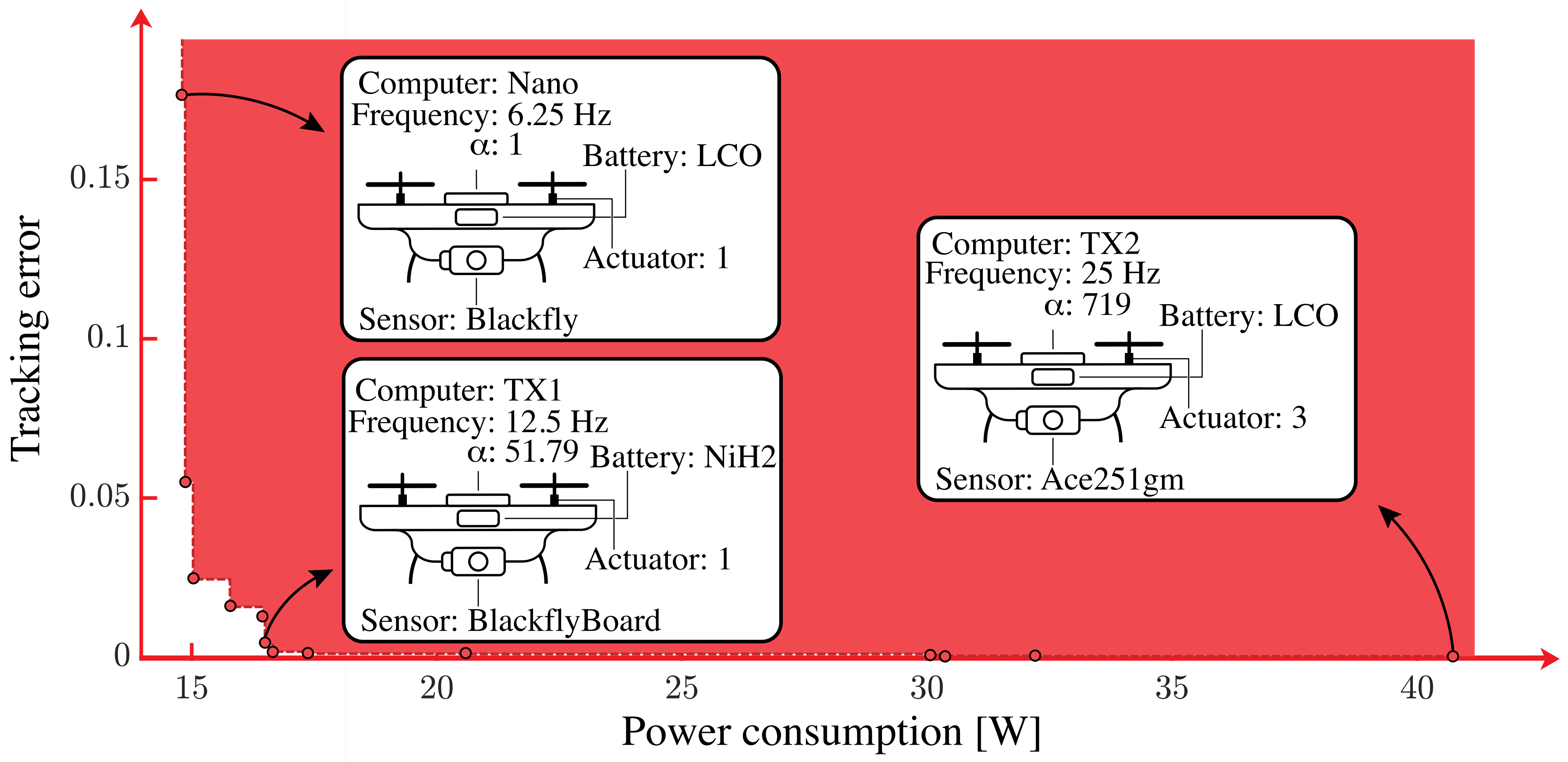

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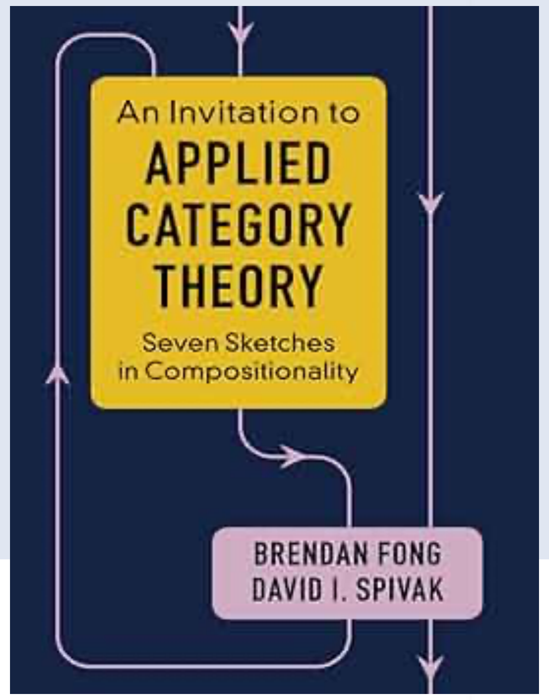

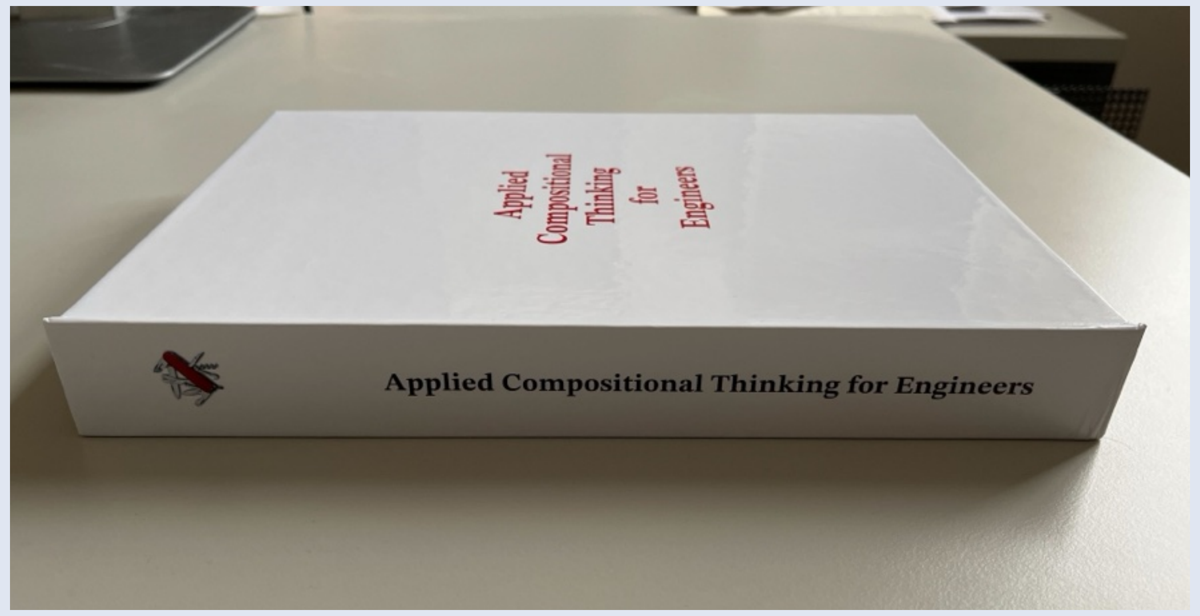

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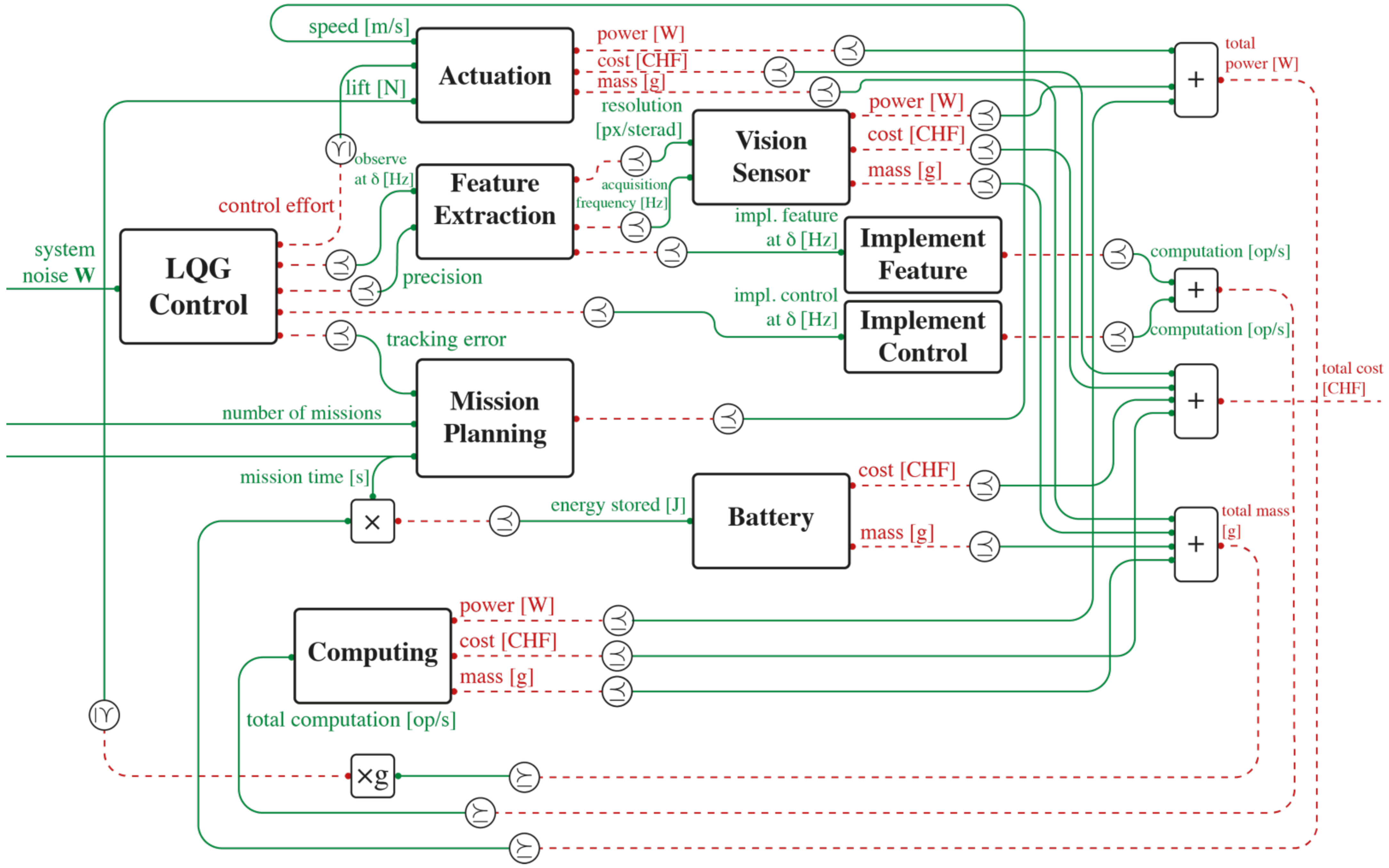

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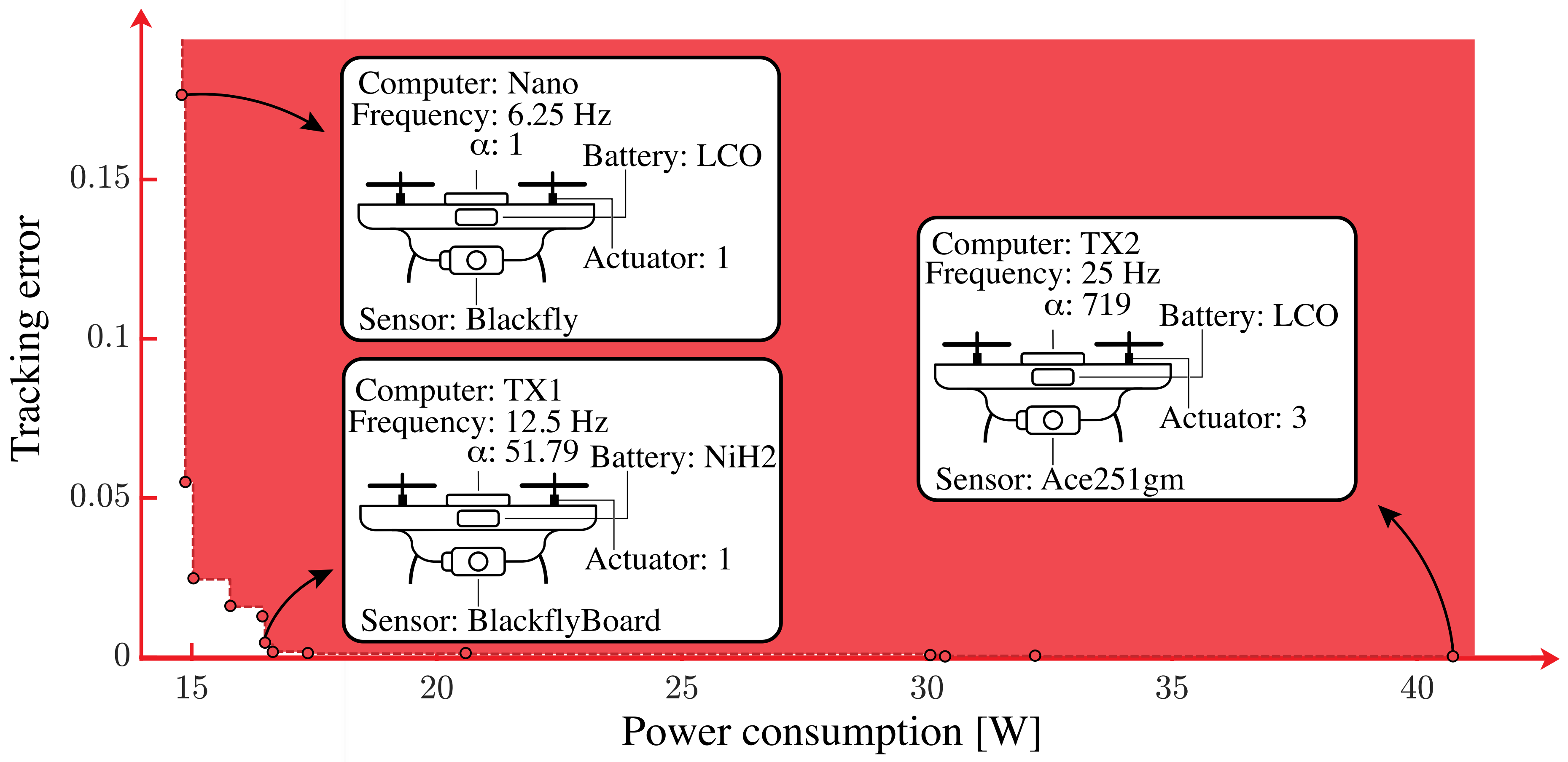
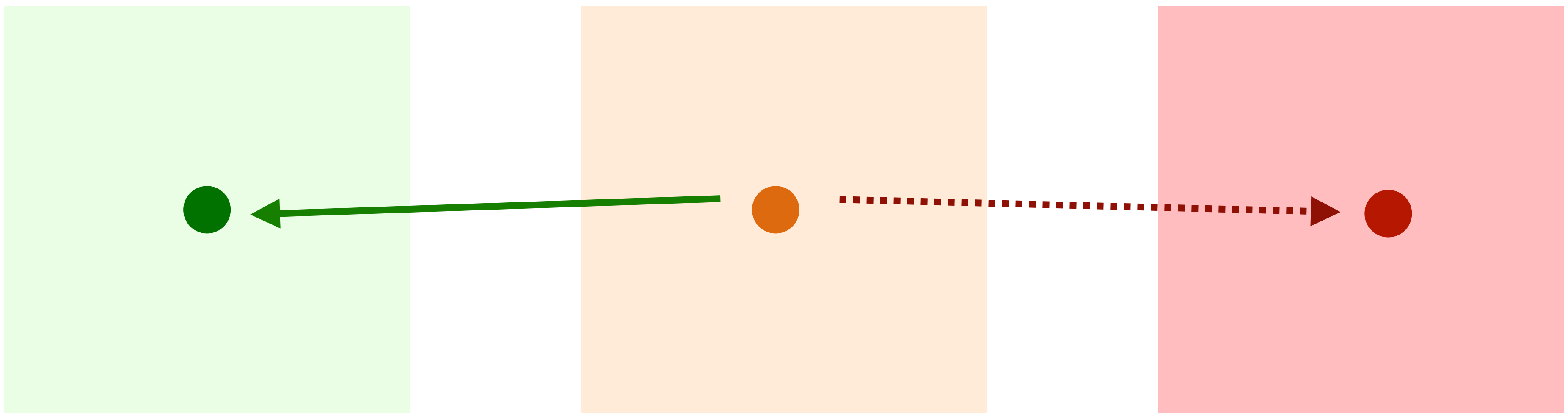


An abstract view of design problems

- Across fields, design or synthesis problems are defined with **three spaces**:
  - **implementation space**: the **options** we can choose from;
  - **functionality space**: what we need to **provide/achieve**;
  - **requirements/costs space**: the **resources** we need to have available;

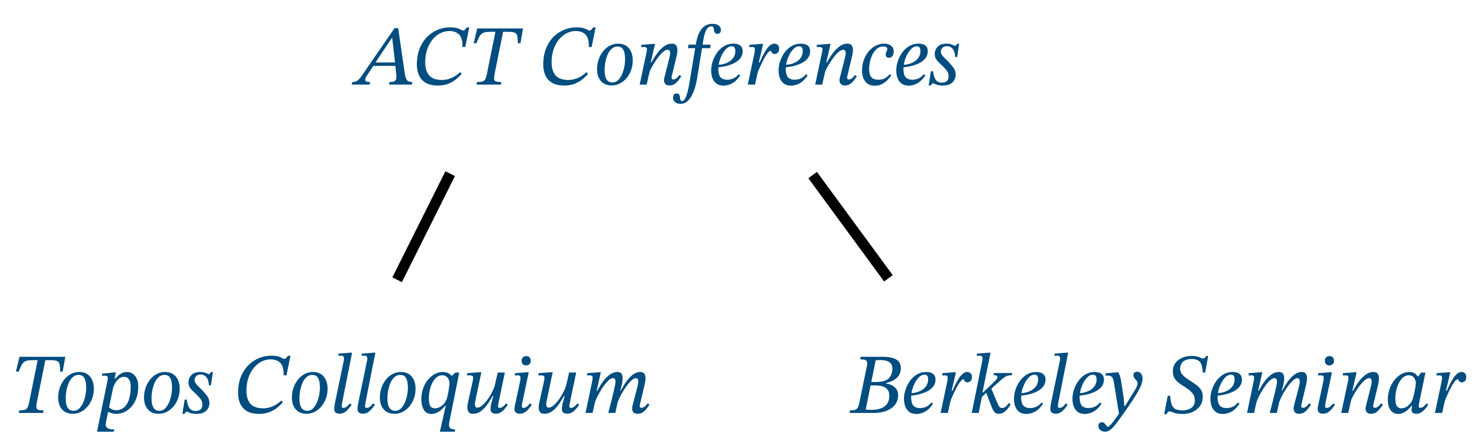


functionality *(provided)*	***implementations***	***costs, resources*** *(required)*
desired behavior	*choices*	*budget*
specifications	*plans*	*requirements*
objectives	*blueprints*	*dependencies*
guarantees	*decision variables*	*assumptions*
conclusions		
"function"	*"form"*	*"function"*

Partially ordered sets model trade-offs, across fields

- Posets model standard costs in engineering $\langle \mathbb{R}_{\geq 0}, \leq \rangle$, $\langle \mathbb{N}, \leq \rangle$
- ... but also enable **richer** cost structures, with **incomparable** elements

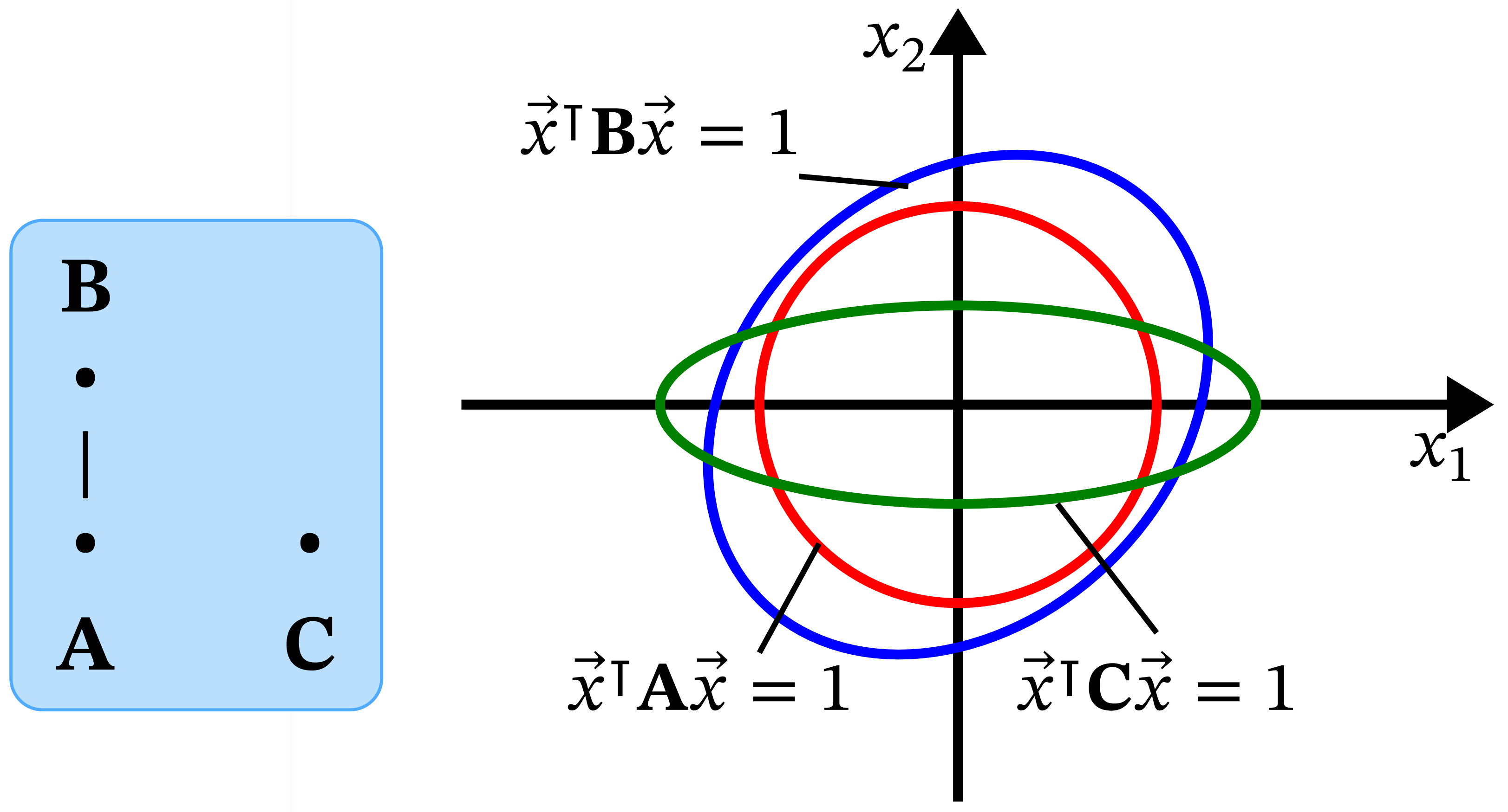
A poset of ACT talks venues



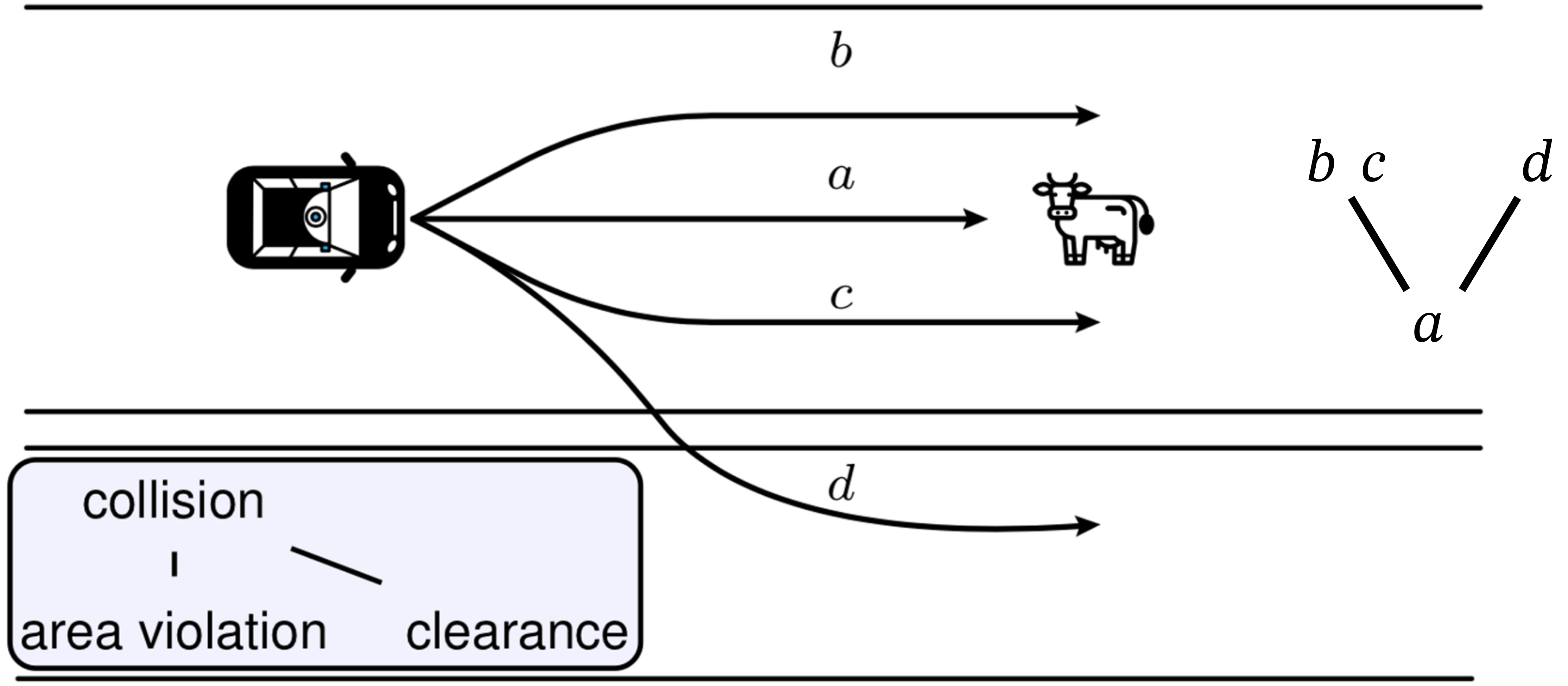
A poset of positive-definite matrices

$$\frac{\mathbf{A} \preceq_{\mathsf{PDM}(n)} \mathbf{B}}{\vec{x}^{\intercal}\mathbf{A}\vec{x} \leq \vec{x}^{\intercal}\mathbf{B}\vec{x} \quad \forall \vec{x} \in \mathbb{R}^{n}}$$

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 3/4 & -1/8 \\ -1/8 & 3/4 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix}$$



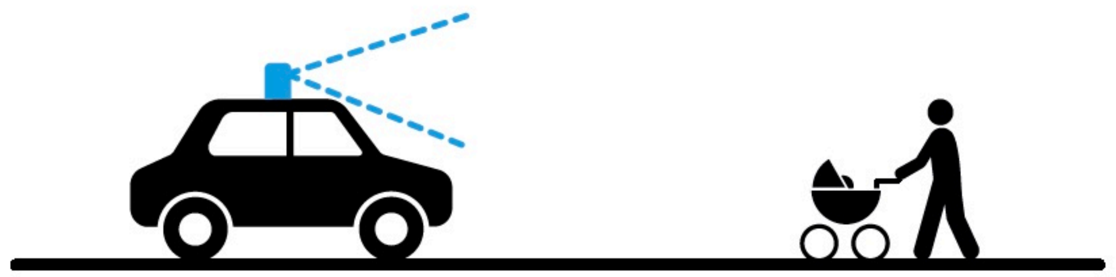
Posets of rules, which induce priorities over behaviors

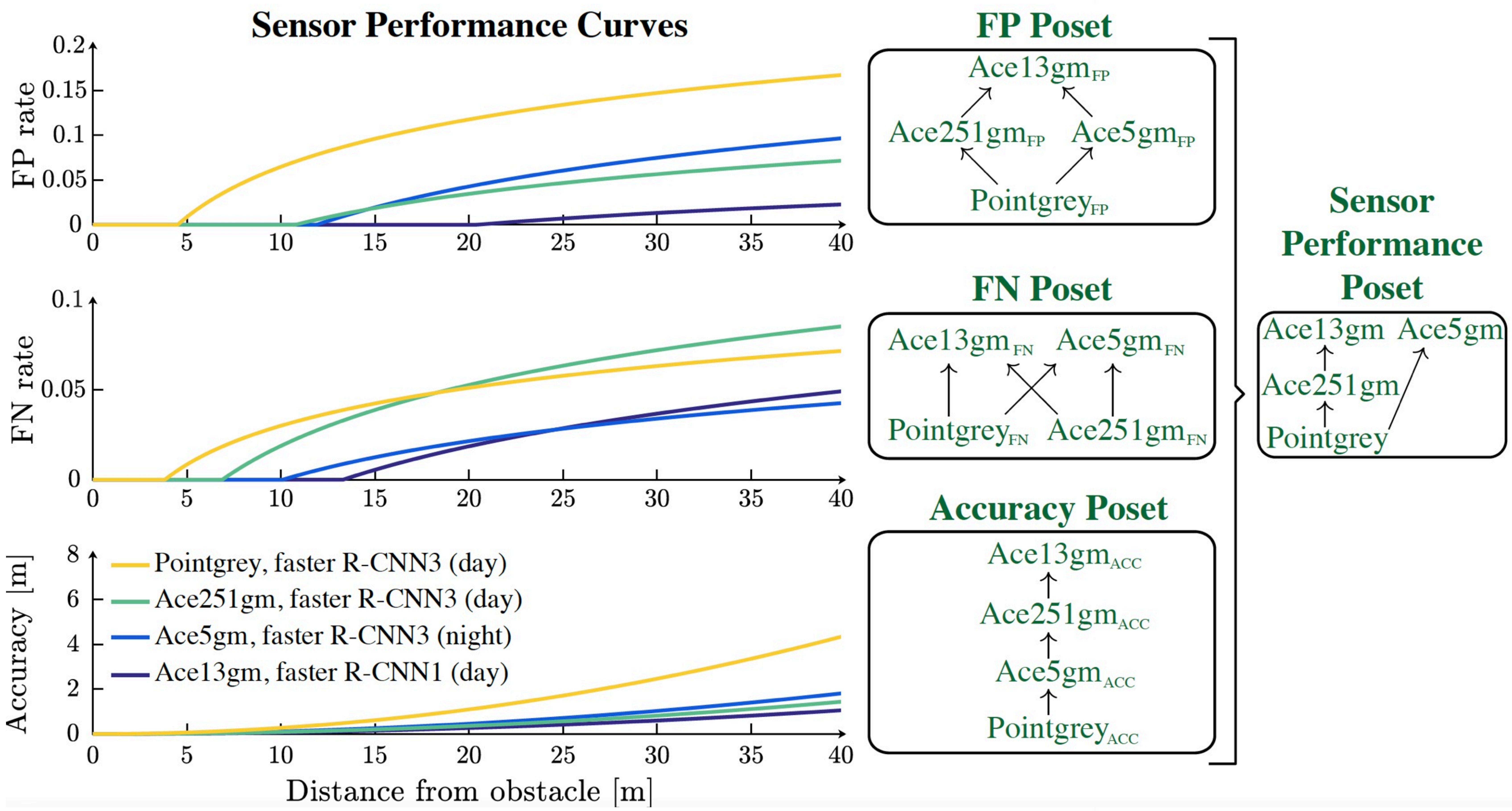


[RAL'22, CDC'22, IROS'21, ECC'21]

Partially ordered sets model trade-offs, across fields

A poset of sensor/algorithm pairs





[CDC'22, IROS'21]

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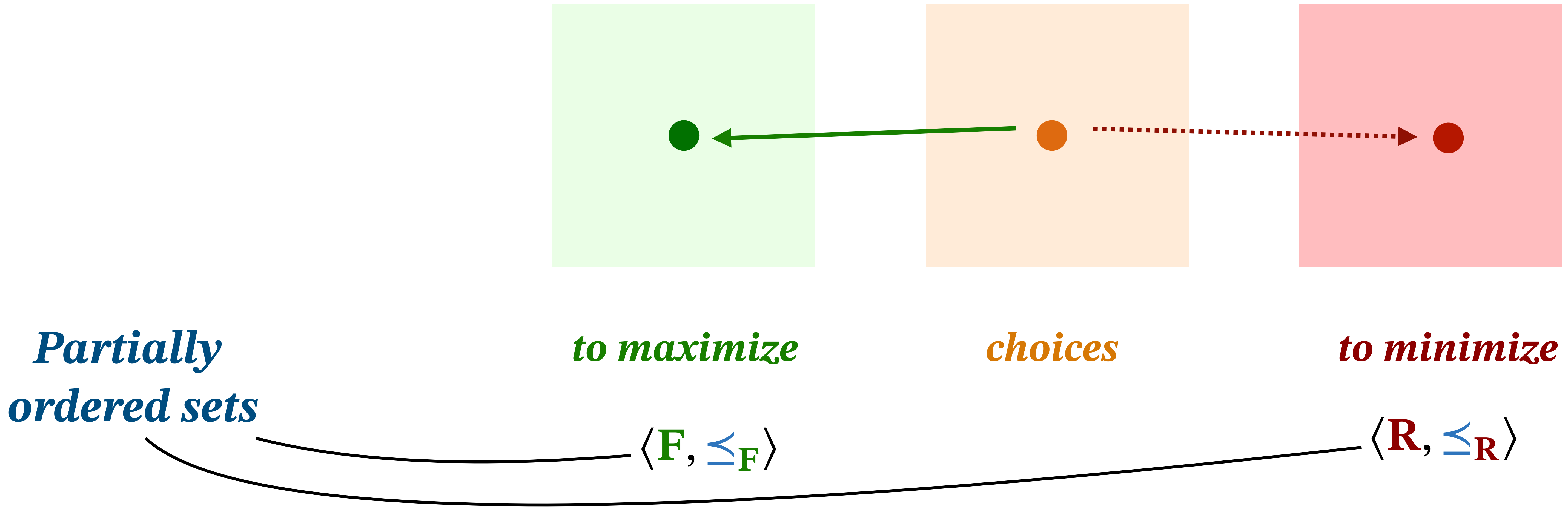
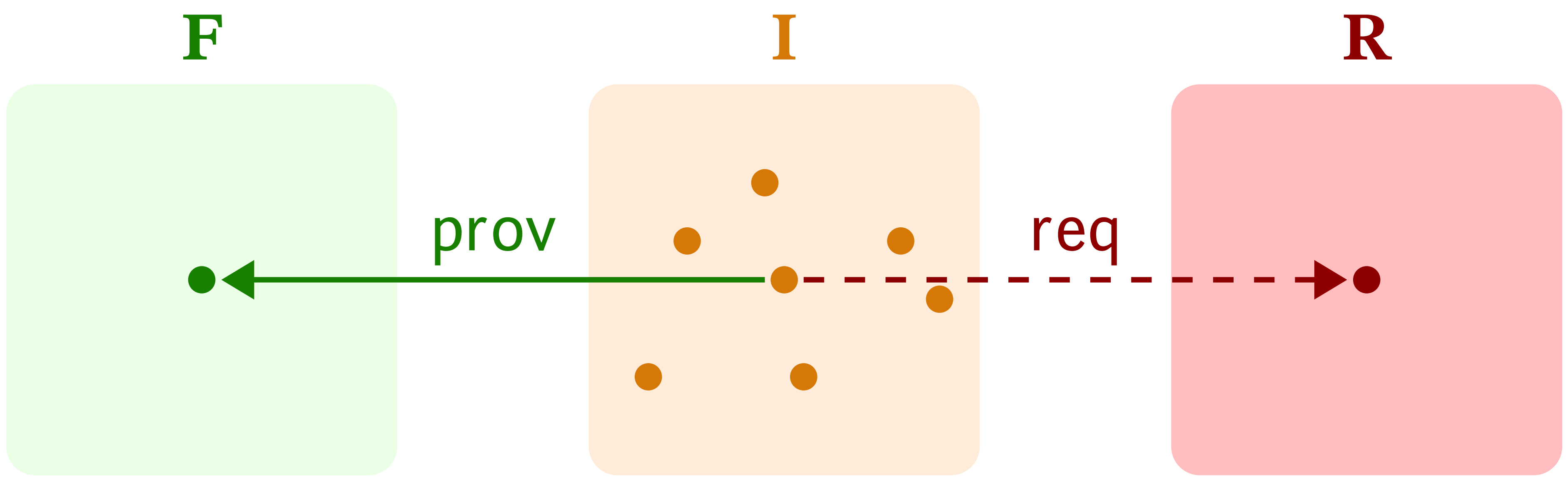


Transparent vs black-box models

- The **"Design Problems with Implementations" model** is a **"transparent" model:**



- **DP** model: **direct feasibility relation** between functionality and resources ("black box") as a monotone map:

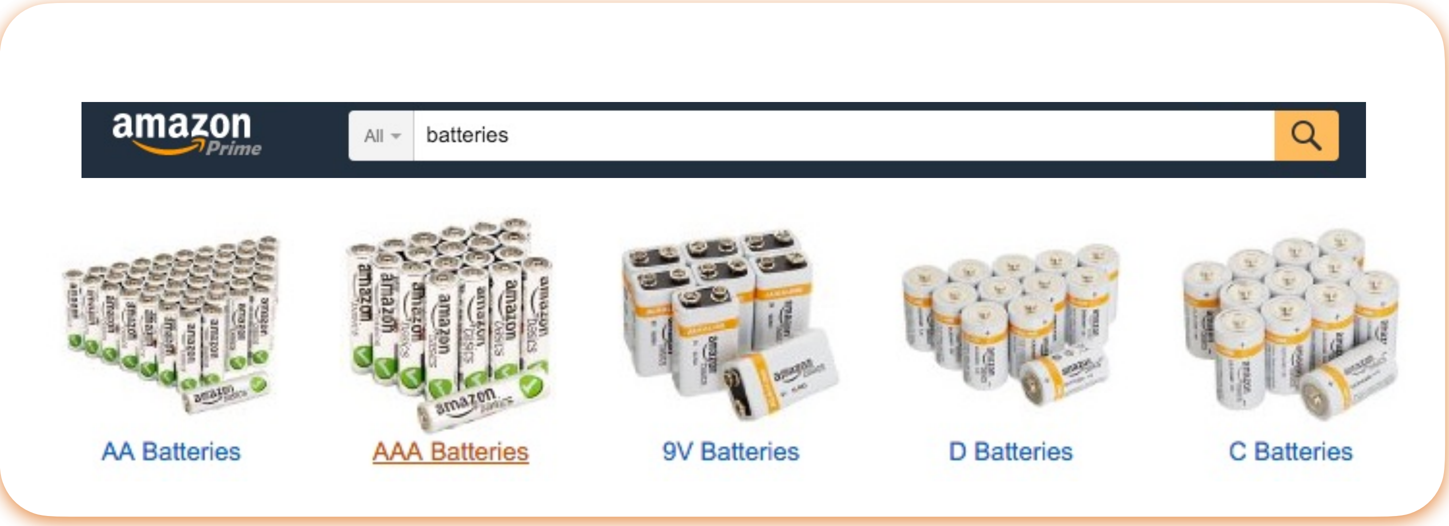
$$\mathbf{d} : \mathbf{F}^{\mathrm{op}} \times \mathbf{R} \to_{\mathbf{Pos}} \mathbf{Bool}$$
$$\langle f^*, r\rangle \mapsto \exists i \in \mathbf{I} : (f \preceq_{\mathbf{F}} \mathtt{prov}(i)) \wedge (\mathtt{req}(i) \preceq_{\mathbf{R}} r)$$

... a "boolean profunctor"

- **Monotonicity**:
  - **Lower functionality** does **not** require **more resources**;
  - More **resources** do not provide **less functionality.**

Co-design enables a rich class of model population techniques

- “**Catalogues**”: off-the-shelf designs.



	Spark	Phantom 3 Std	Phantom 4 Adv	Phantom 4 Pro	Mavic	Inspire
Flight time	16 mins	25 mins	30 mins	30 mins	27 mins	27 mins
Top Speed	31 mph (50 km/h)	36 mph (58 km/h)	45 mph (72 km/h)	45 mph (72 km/h)	40 mph (65 km/h)	58 mph (94 km/h)
Range	1.2 miles (2 km)	0.6 miles (1 km)	4.3 miles (7 km)	4.3 miles (7 km)	4.3 miles (7 km)	4.3 miles (7 km)
Camera	12-MP stills 1080p video	12-MP stills 2704 x 1520p video	20-MP stills 4K 60fps video	20-MP stills 4K 60fps video	12-MP stills 4K video	20.8-MP stills 4K/5K video
Size	5.6 x 5.6 x 2.1 in (14.3 x 14.3 x 5.5 cm)	13.8 in diagonal (350 mm)	13.8 in diagonal (350 mm)	13.8 in diagonal (350 mm)	13.2 in diagonal (350 mm)	16.8 x 12.5 x 16.7 in (42.7 x 31.7 x 42.5 cm)
Takeoff weight	11.6 oz (330 g)	2.6 lb (1.2 kg)	3 lb (1.4 kg)	3 lb (1.4 kg)	1.6 lb (743 kg)	8.8 lb (4 kg)
Other features	Follow me, Return home, Obstacle avoidance, FPV	Follow me, Return home	Follow me, Return home, Obstacle avoidance	Follow me, Return home, 3 Direction Obstacle avoidance	Follow me, Return home, Obstacle avoid-ance, folding arms	Obstacle avoidance, Spotlight Pro/Broadcast/ Composition mode
Price	US$499	US$499	US$1,349	US$1,499	US$999	US$2,999 ($6,198 with camera/gimbal)

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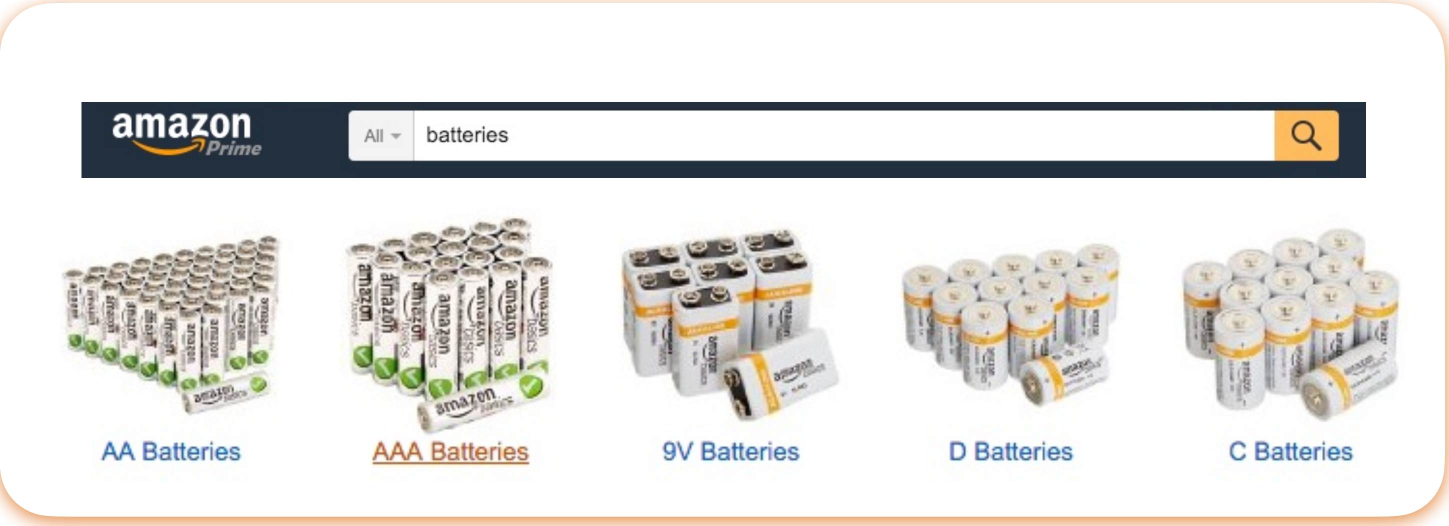

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- “**First-principles**”: analytical relations.

mission energy ≥ mission duration x *power consumption*

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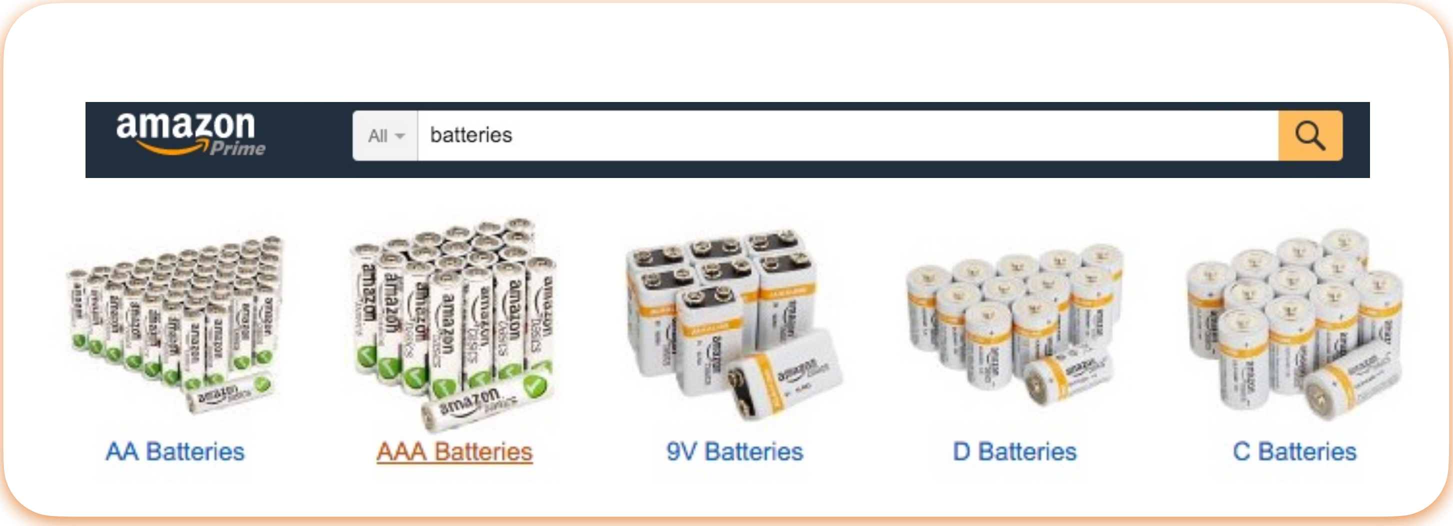

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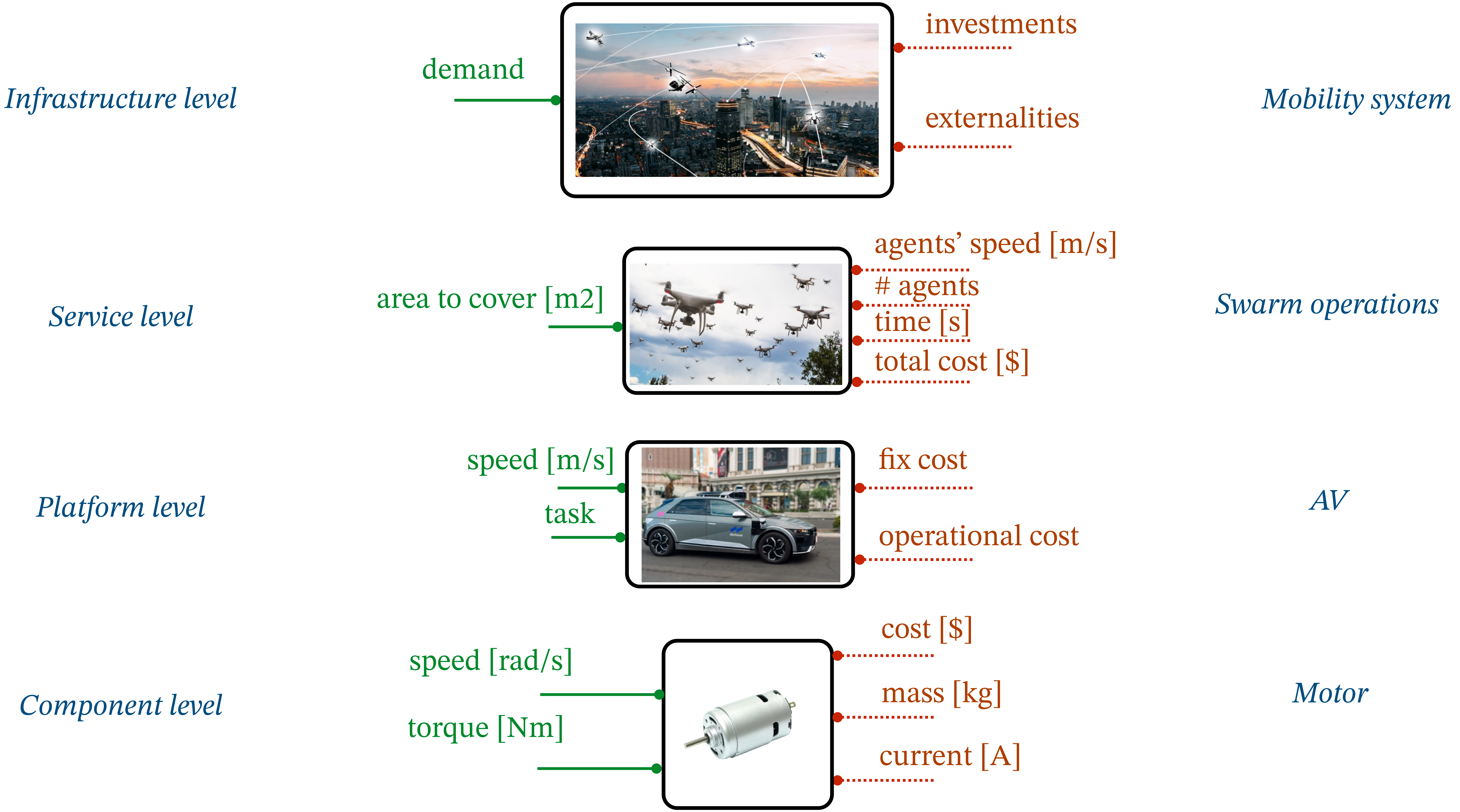
- "**First-principles**": analytical relations.

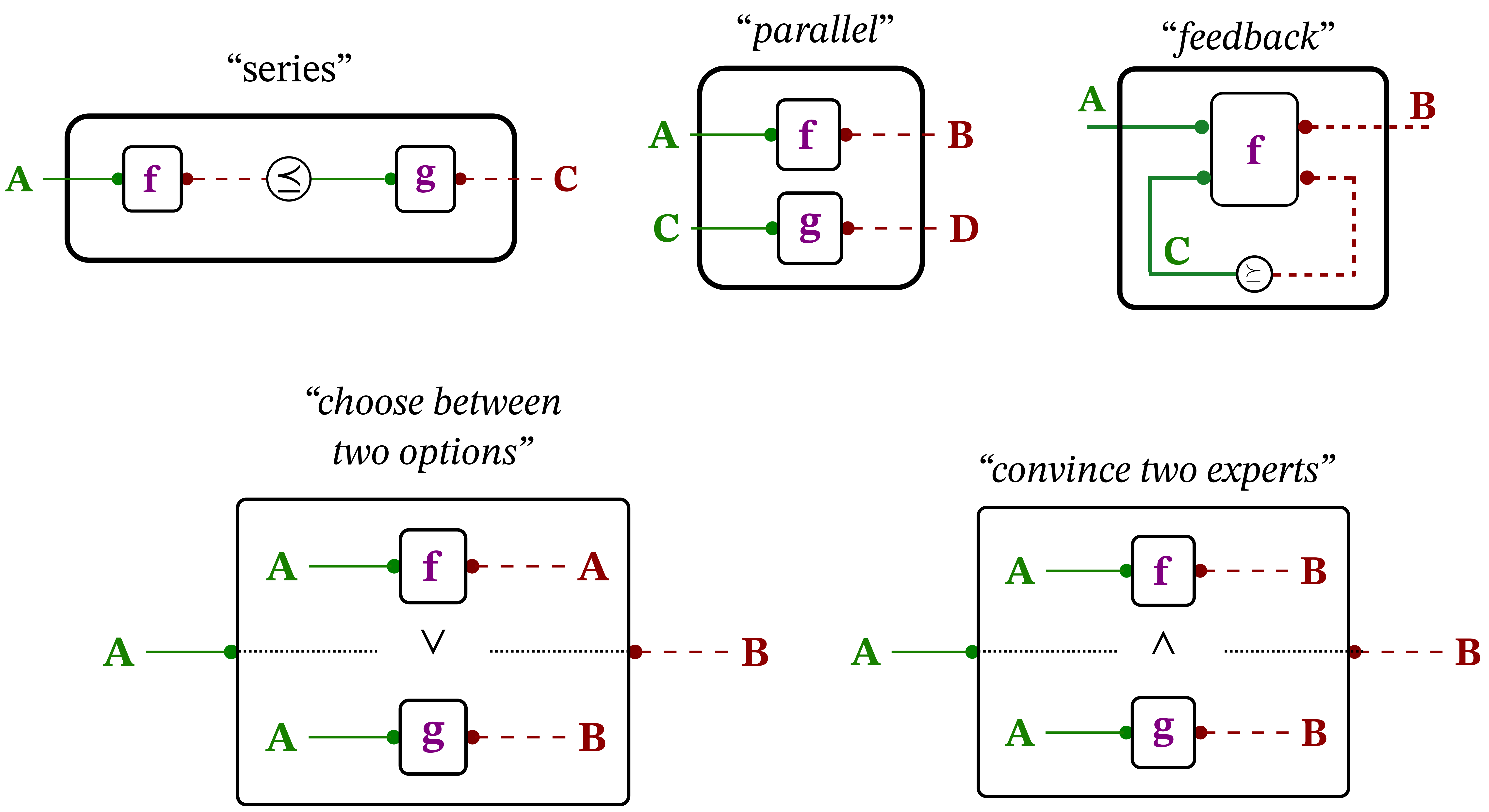
mission energy ≥ mission duration x *power consumption*

- "**Data-driven**", "**on-demand**"
  - The optimization will only ask for a **sequence** of data points. The model is constructed **incrementally**.
  - Opens the door to **experiments**, black-box **simulations**, solutions of **optimization problems**.

Design problems arise naturally in many domains, across scales



Design problems can be composed in various ways, preserving properties



- The **composition** of any two **DPs** returns a **DP** (closure)
- Very practical tool to **decompose** large **problems** into **subproblems**

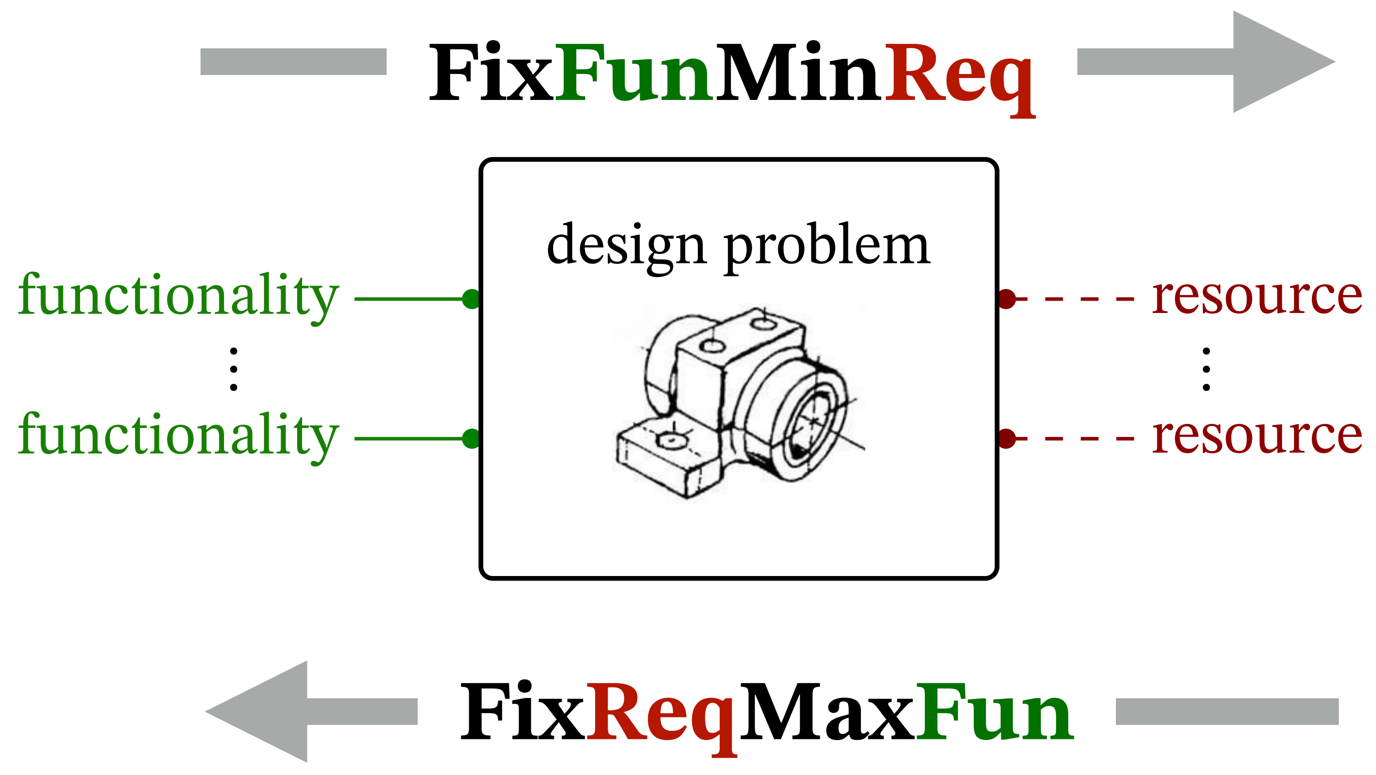
*There is a category **DP** which is traced monoidal, and locally posetal*

✔ ***Formal***
Compositional/hierarchical

Multiple queries from the same design problem

- Two basic design queries are:

Given the functionality to be provided,
what are the **minimal resources** required?



Given the resources that are available, what is
the **maximal functionality** that can be provided?

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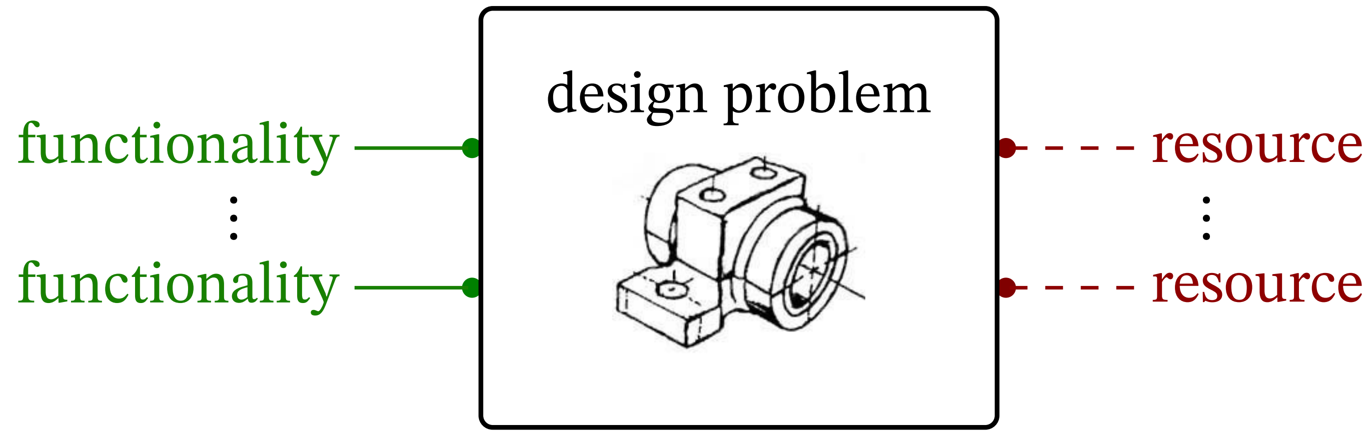
FixFunMinReq



FixReqMaxFun

Given the resources that are available, what is the **maximal functionality** that can be provided?

- The two problems are **dual**

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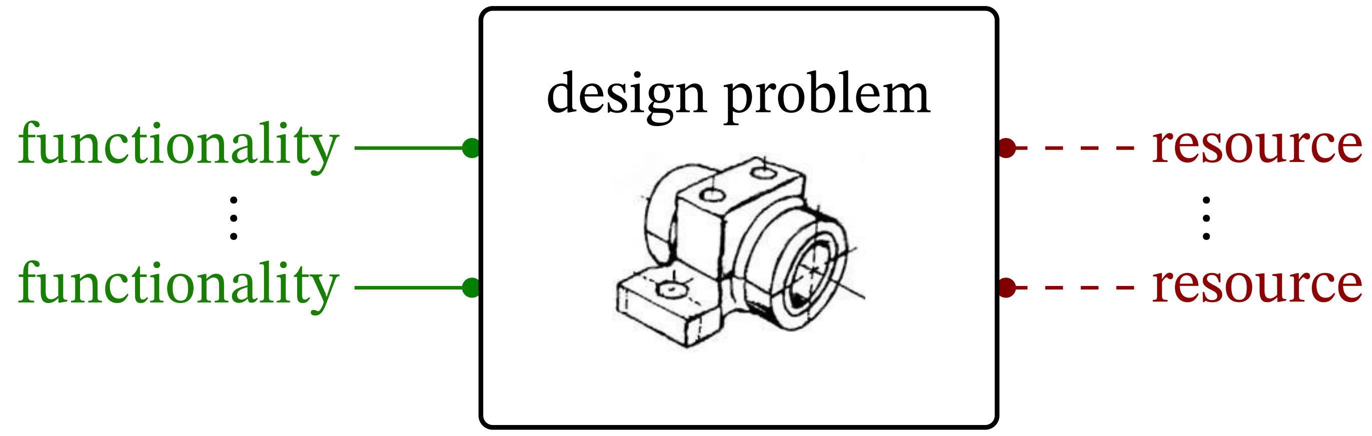
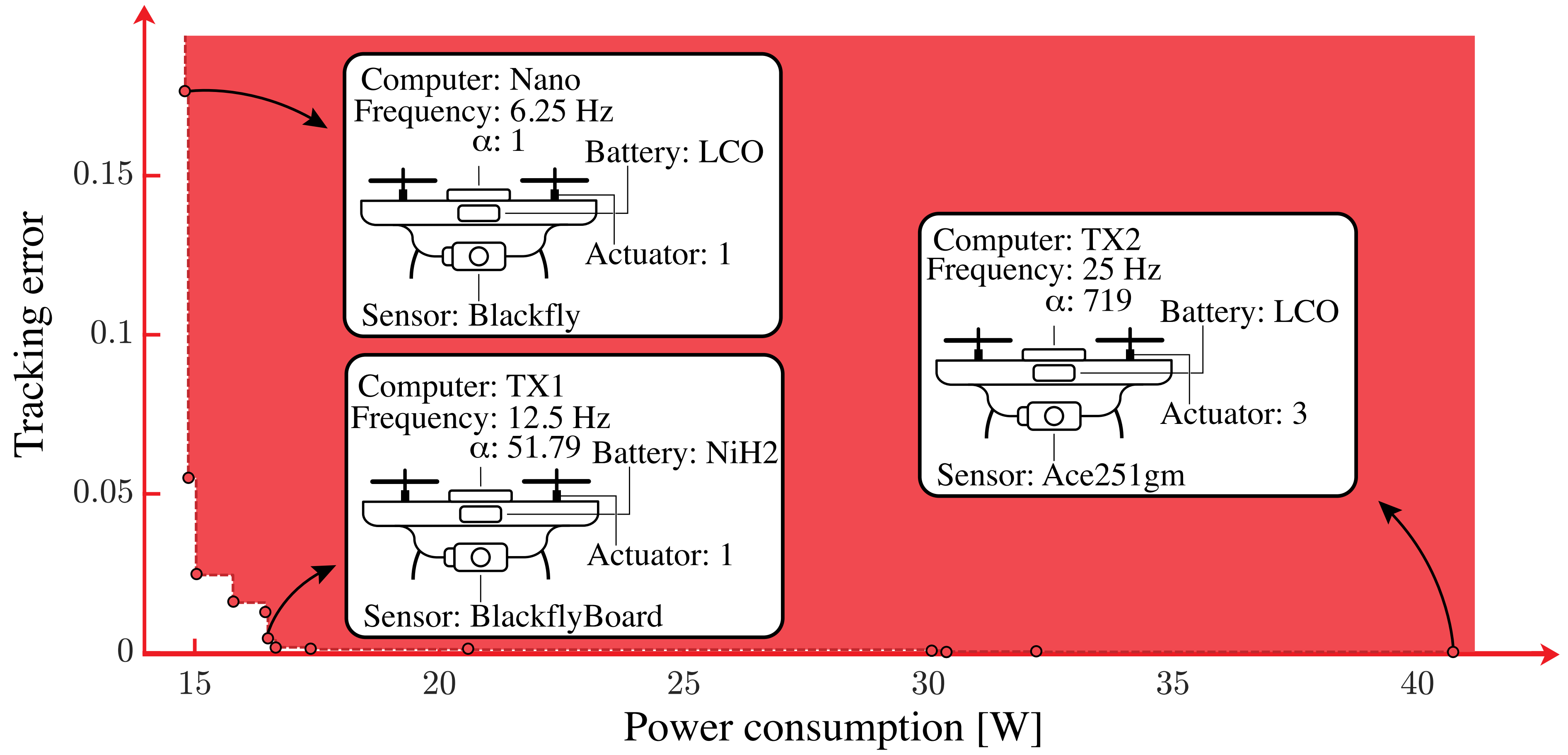
FixFunMinReq



FixReqMaxFun

Given the resources that are available, what is
the **maximal functionality** that can be provided?

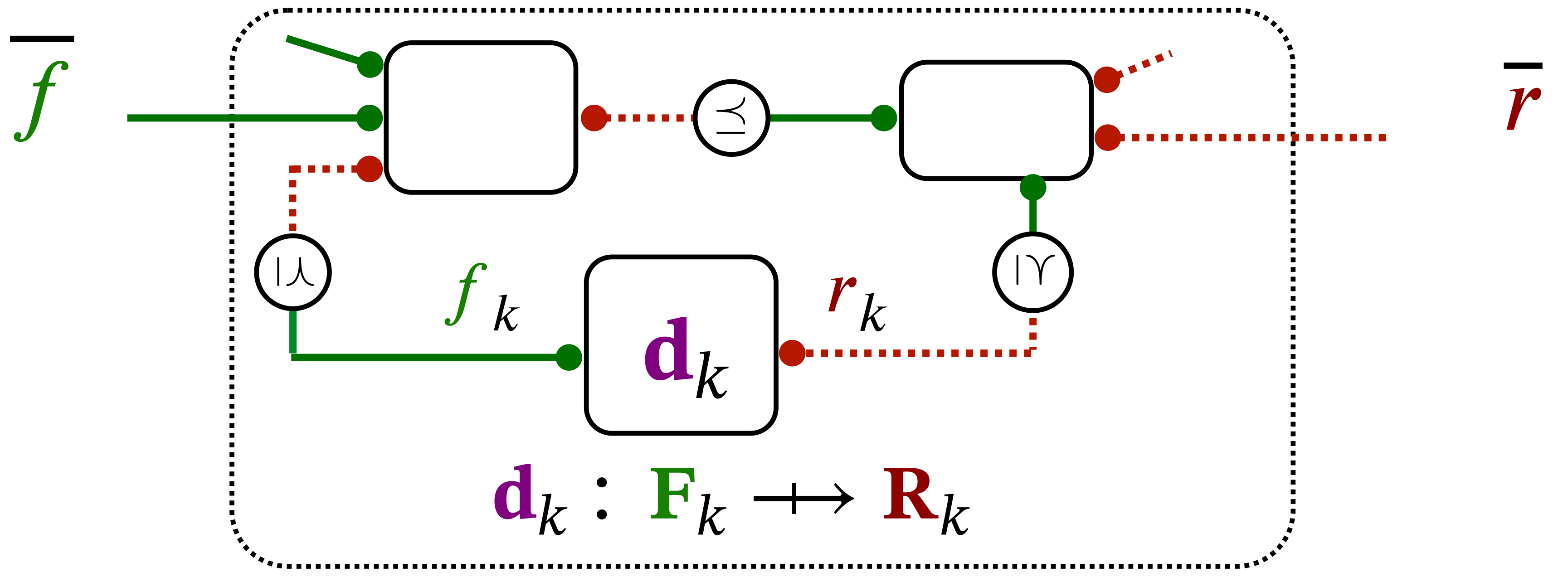
- We are looking for:
  - A map from functionality to **upper sets** of feasible resources: $h : \mathbf{F} \to \mathcal{U}\mathbf{R}$
  - A map from functionality to **antichains** of minimal resources: $h : \mathbf{F} \to \mathcal{A}\mathbf{R}$



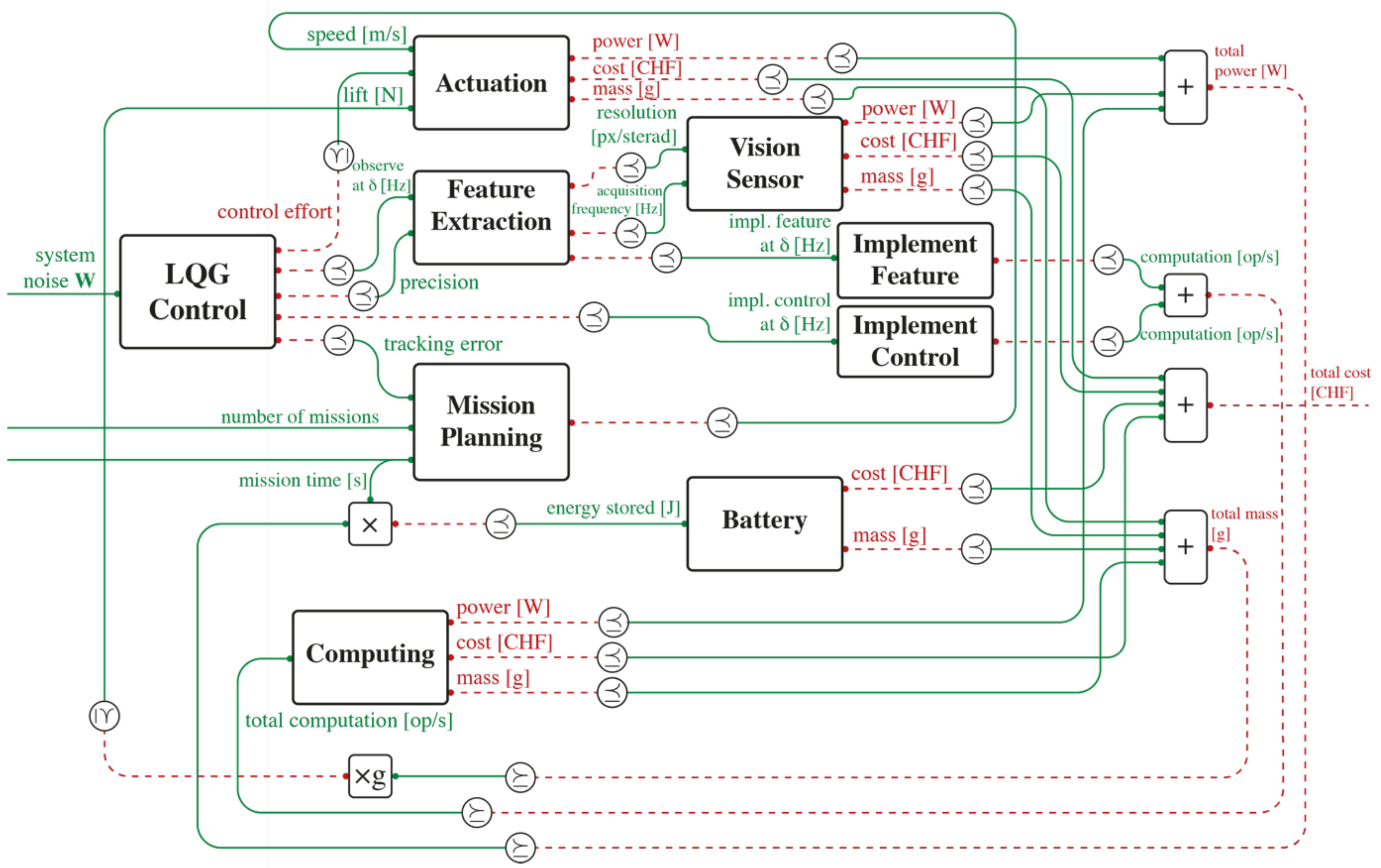
Optimization semantics

- This is the semantics of **Fix****Fun****Min****Req** as a **family of optimization problems.**

chosen by user

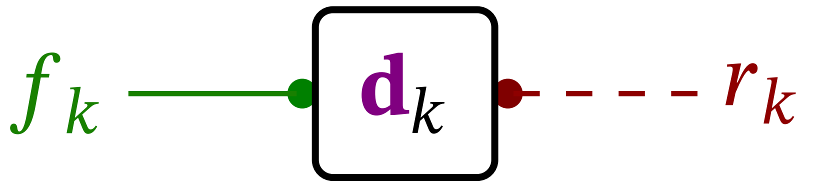


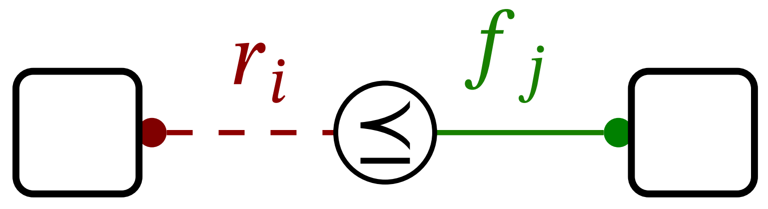
to minimize



variables $f_k \in \langle \mathbf{F}_k, \preceq_{\mathbf{F}_k} \rangle$ $r_k \in \langle \mathbf{R}_k, \preceq_{\mathbf{R}_k} \rangle$

constraints *for each node:* *for each edge:*





$\mathbf{d}_k(f_k^*, r_k) = \top$

component feasibility

$r_i \preceq f_j$

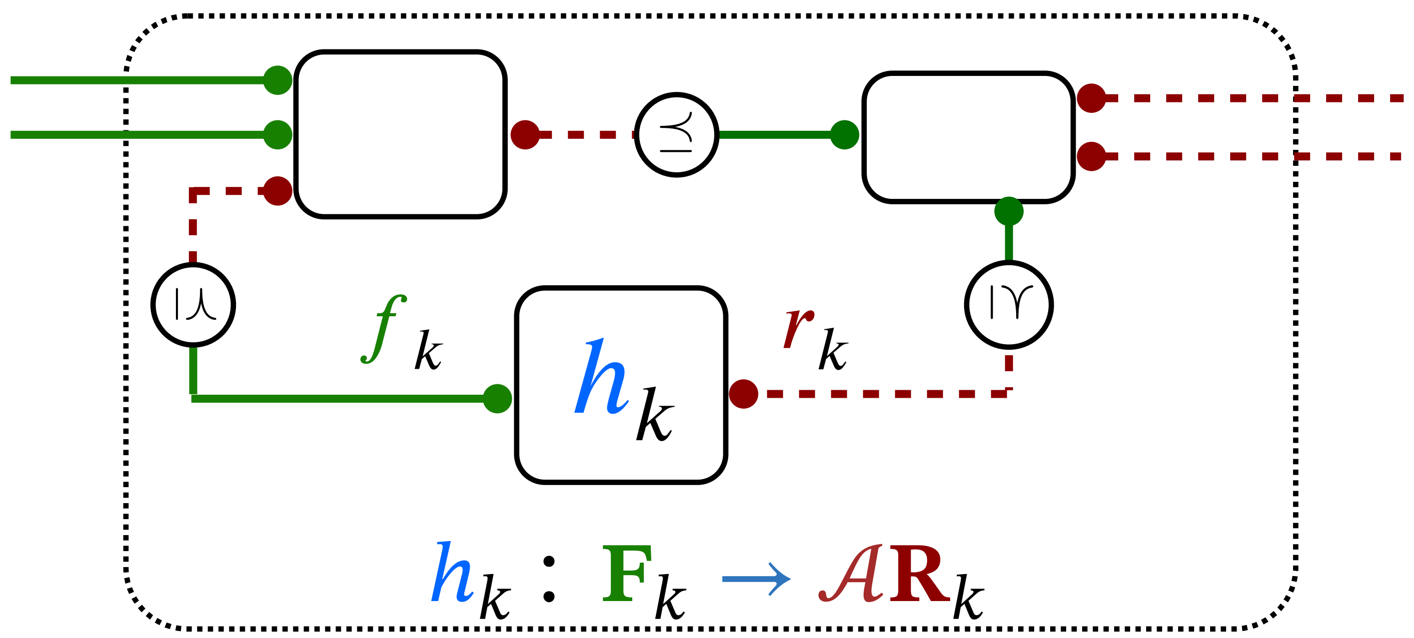
co-design constraint

objective $\operatorname{Min}_{\preceq} \bar{r}$

! **not convex**
! **not differentiable**
! **not continuous**
! **not even defined on continuous spaces**

Compositional solution of design problem queries

- Suppose that we are given the map $h_k : \mathbf{F}_k \to \mathcal{A}\mathbf{R}_k$ for all nodes in the co-design graph



- Can we find the map $h : \mathbf{F} \to \mathcal{A}\mathbf{R}$ for the entire graph?

Computationally tractable

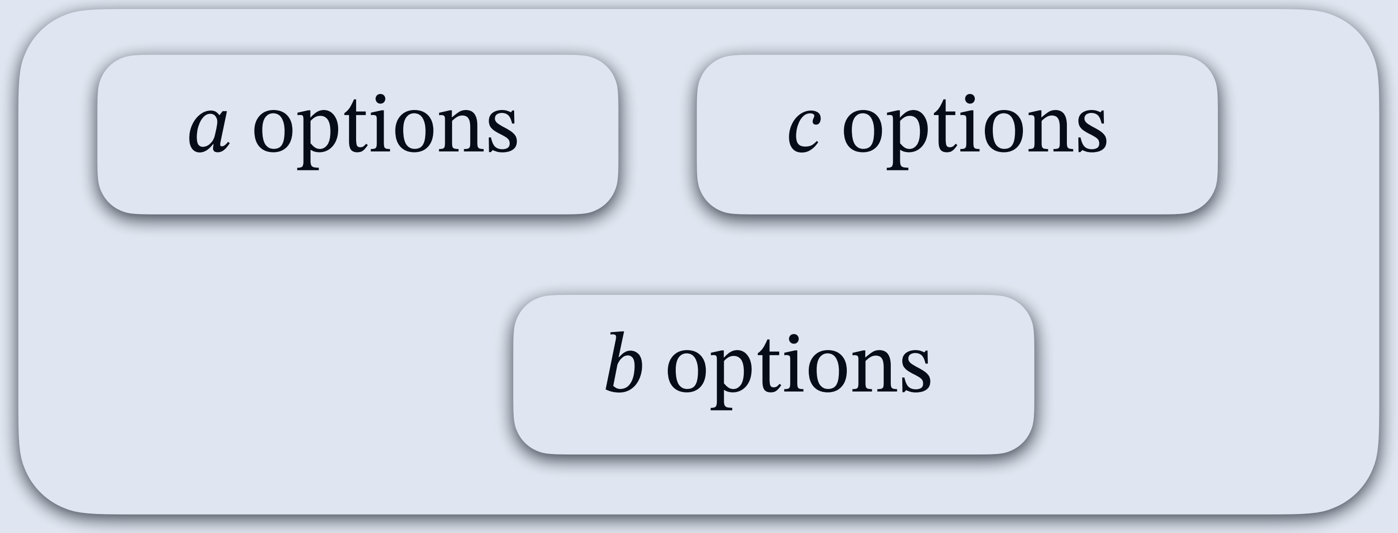
- **Compositional approach**: just need to work out the composition formulas for all operations

solution(**composition**(a, b)) = **composition**(**solution**(a), **solution**(b))

... a functor between a category of problems and one of solutions

- The set of **minimal** feasible resources can be obtained as the **least fixed point** of a monotone function in the space of anti-chain

- We have a **complete solution**: guaranteed to find the set of **all** optimal solutions (if empty, **certificate of infeasibility**)
- The complexity is **not combinatorial in the number of options** for each component
- The complexity depends on the **complexity of the interactions**: the co-design **constraints**



$O(a + b + c)$

A new category of upper sets

Definition (Category $\mathbf{Pos}_U$)
The category $\mathbf{Pos}_U$ consists of:

1. *Objects*: objects are posets;
2. *Morphisms*: given objects $X, Y \in \mathrm{Ob}_{\mathbf{Pos}_U}$, morphisms from $f : X \to Y$ are monotone maps of the form $f^\star : X \to_{\mathbf{Pos}} UY$.
3. *Composition of morphisms*: Given morphisms $f : X \to Y$, $g : Y \to Z$, their composition $f \,⨾\, g : X \to Z$ is given by

$$\begin{aligned} (f \,⨾\, g)^\star : X &\to_{\mathbf{Pos}} UZ \\ x &\mapsto \bigcup_{y \in f^\star(x)} g^\star(y); \end{aligned}$$

4. *Identity morphism*: given an object $X \in \mathrm{Ob}_{\mathbf{Pos}_U}$, the identity morphism $\mathrm{id}_X : X \to X$ is given by the application of the upper closure operator:

$$\mathrm{id}_X^\star(x) := \uparrow\{x\}.$$

From problems to solutions

Lemma 7.23. There is a functor

$$\mathsf{FixFunMinRes} : \mathbf{DP} \to \mathbf{Pos}_U \tag{50}$$

that maps:

1. An object (poset) in $\mathbf{DP}$ to the same object (poset) in $\mathbf{Pos}_U$.
2. A morphism $\mathbf{e} \in \mathrm{Hom}_{\mathbf{DP}}(\mathbf{F}; \mathbf{R})$ to the morphism $H_{\mathbf{e}} \in \mathrm{Hom}_{\mathbf{Pos}_U}(\mathbf{F}; \mathbf{R})$, where:

$$\begin{aligned} H^{\star}_{\mathbf{e}} : \mathbf{F} &\to_{\mathbf{Pos}} U\mathbf{R} \\ f &\mapsto \{r \in \mathbf{R} \mid \mathbf{e}(f^*, r)\}. \end{aligned}$$

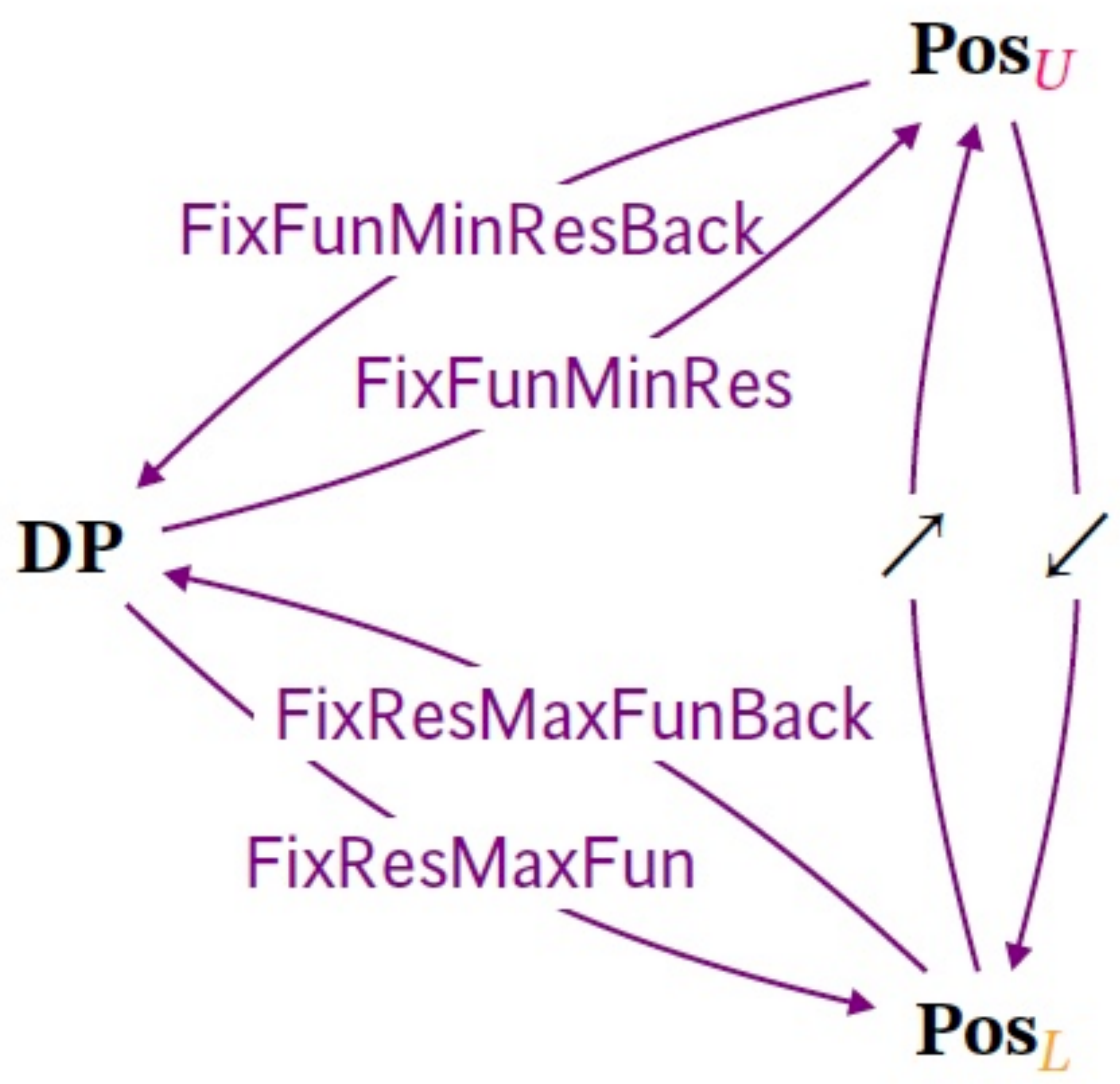
Lemma 7.24. There is a functor

$$\mathsf{FixResMaxFun} : \mathbf{DP} \to \mathbf{Pos}_L$$

which maps:

1. An object (poset) of $\mathbf{DP}$ to the same object (poset) in $\mathbf{Pos}_L$.
2. A morphism $\mathbf{e} \in \mathrm{Hom}_{\mathbf{DP}}(\mathbf{F}; \mathbf{R})$ to the morphism $K_{\mathbf{e}} \in \mathrm{Hom}_{\mathbf{Pos}_L}(\mathbf{R}; \mathbf{F})$, where:

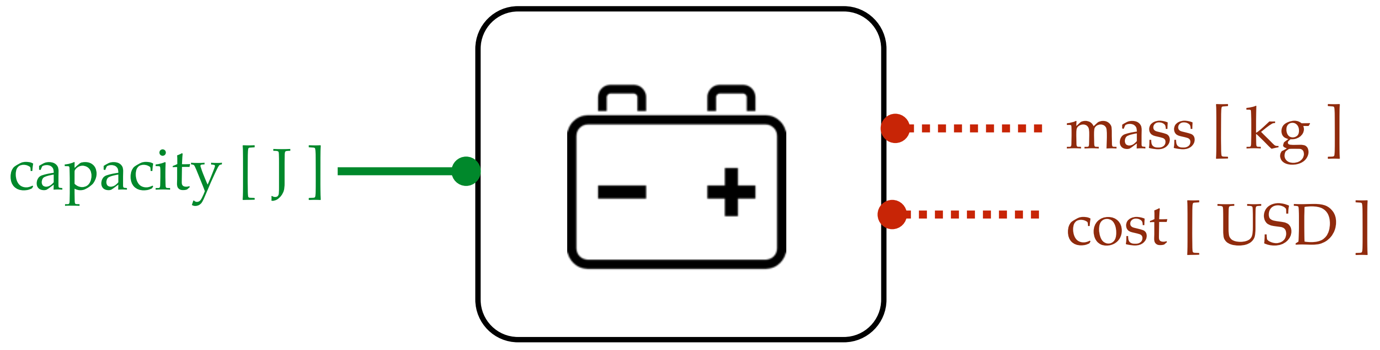
$$\begin{aligned} K^{\star}_{\mathbf{e}} : \mathbf{R} &\to_{\mathbf{Pos}} L\mathbf{F} \\ r &\mapsto \{f \in \mathbf{F} \mid \mathbf{e}(f^*, r)\}. \end{aligned}$$



Interesting question: enriching in performance?

User-friendly interfaces

- "**Catalogues**": already available designs

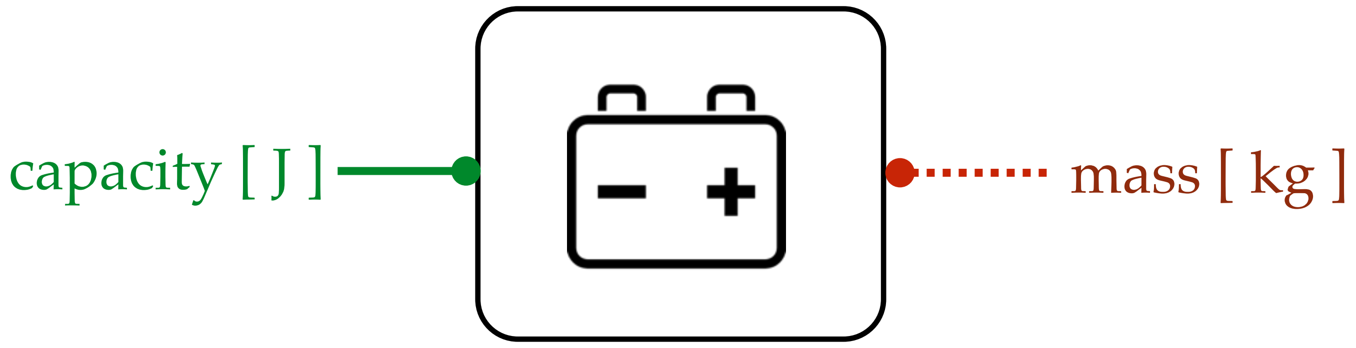



```
catalogue {
    provides capacity [J]
    requires mass [g]
    requires cost [USD]

    500 kWh ⟻ model1 ⟼ 100 g, 10 USD
    600 kWh ⟻ model2 ⟼ 200 g, 200 USD
    600 kWh ⟻ model3 ⟼ 250 g, 150 USD
    700 kWh ⟻ model4 ⟼ 400 g, 400 USD
}
```

... and a solver

- "**First-principles**": analytical relations.




```
mcdp {
    provides capacity [J]
    requires mass [kg]

    specific_energy_Li_Ion = 500 Wh / kg

    required mass >= provided capacity / specific_energy_Li_Ion
}
```

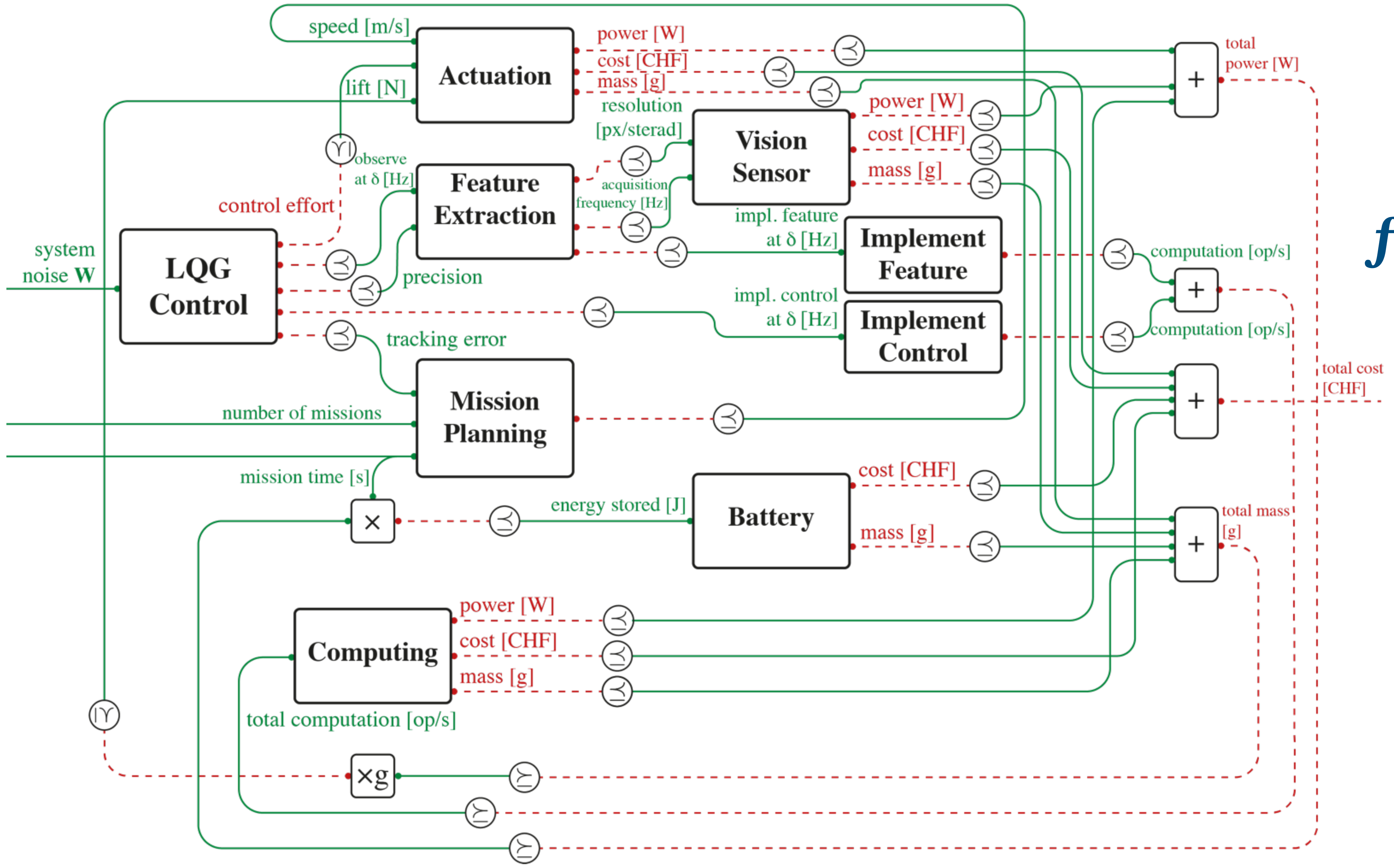

```
mcdp {
    provides lift [N]
    requires power [W]
    c = 10.0 W/N²
    required power ≽ c · provided lift²
}
```

A systematic process for task-driven co-design of complex systems

- A new approach to **co-design** designed to work **across fields** and **across scales**.
- What we have seen:
  - Defining **“design problems”** for **components.**
  - Modeling **co-design constraints** in a complex **system**.
  - **Efficient** solution to design queries.

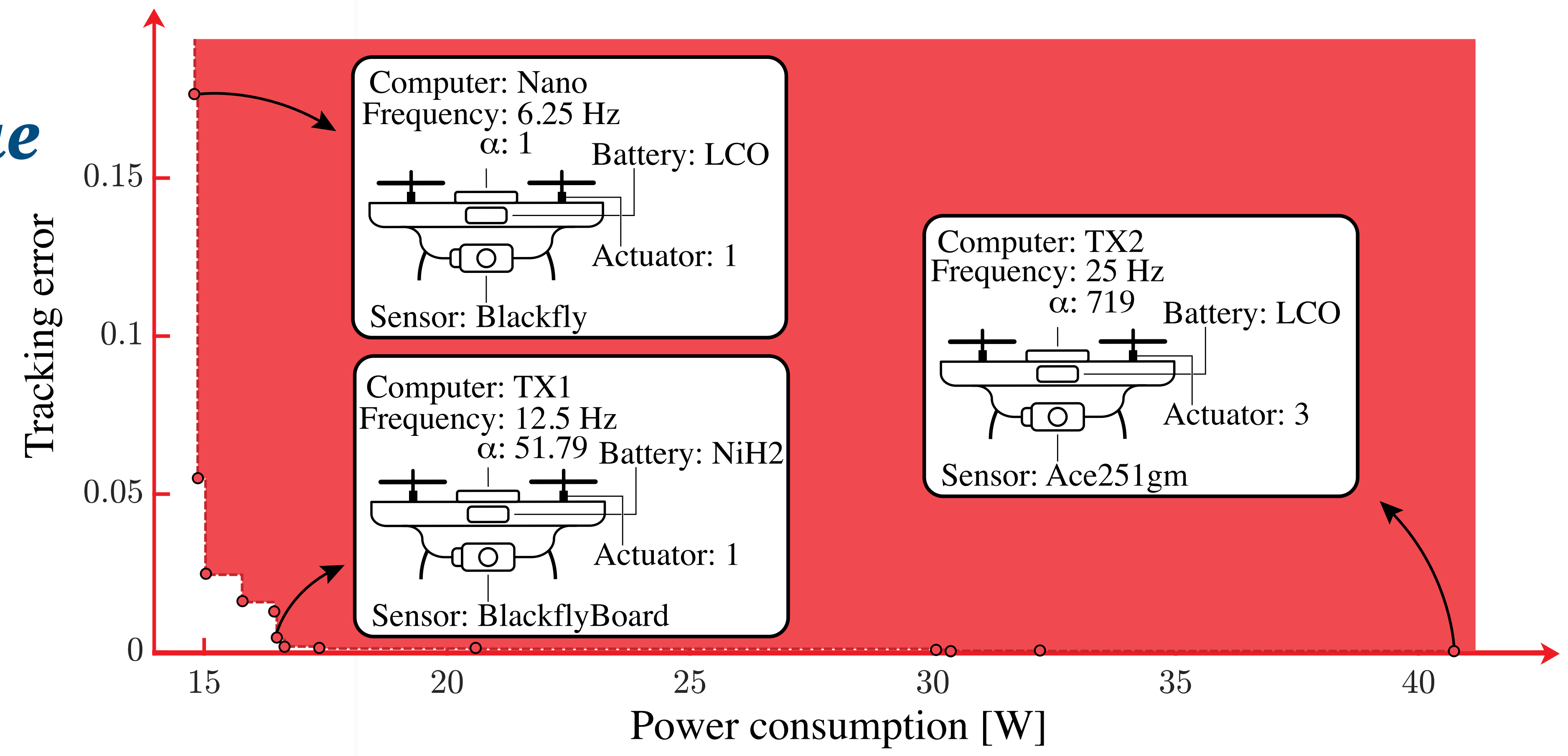
- **Modeling** approach
- Actual **implementation**:
  - Coming up with the skeleton/diagram
  - Populating the models

“Co-design diagram for a drone”



optimization for a search-and-rescue task

Pareto front of optimal designs



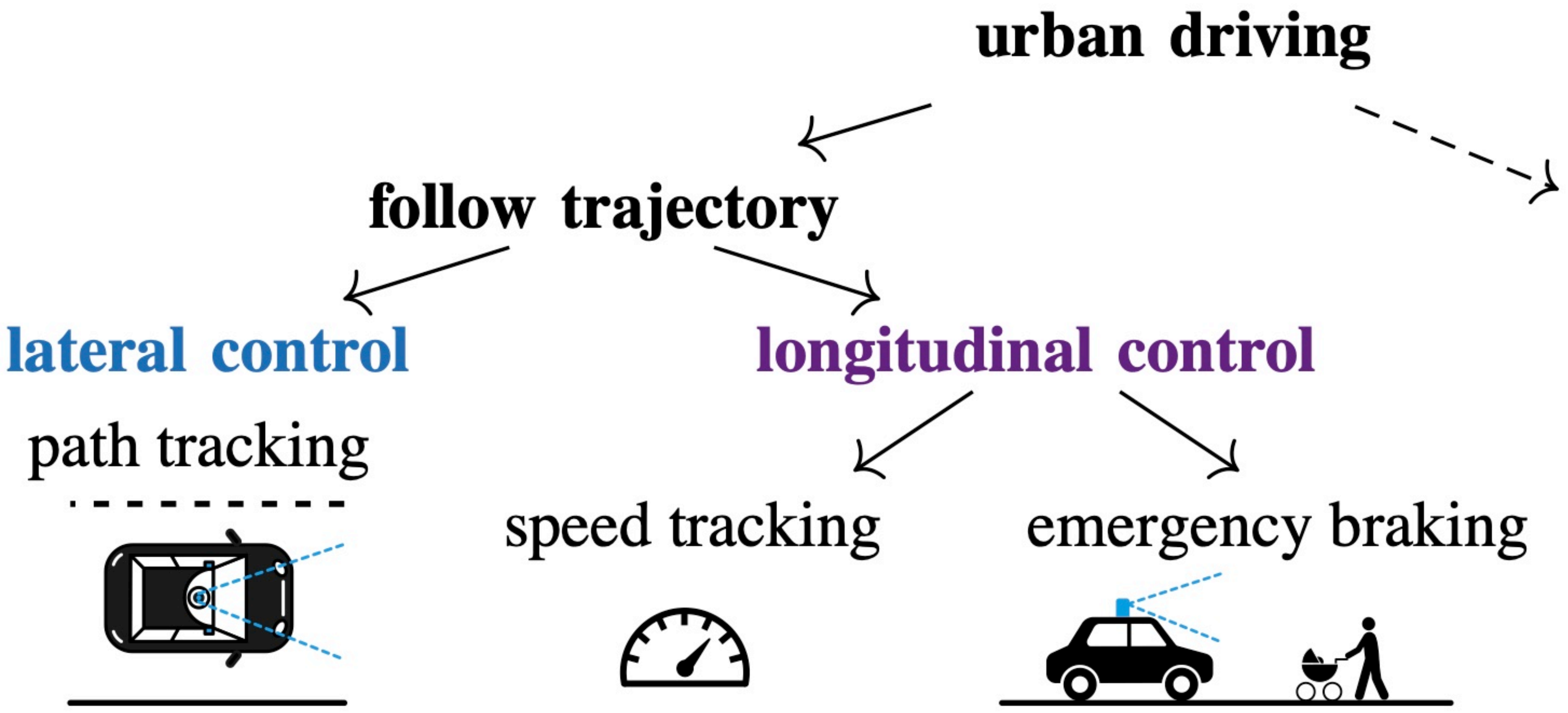
A systematic process for task-driven co-design of complex systems

- A **systematic modeling approach**:
  - **Define the task -** *what do we need to do?*

urban driving

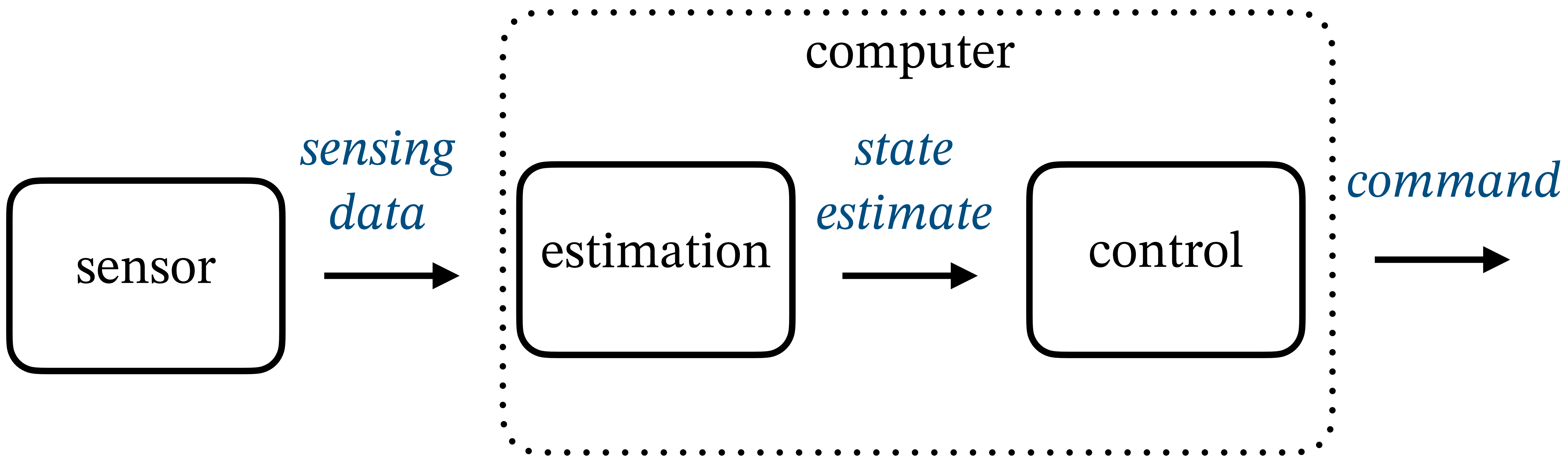
A systematic process for task-driven co-design of complex systems

- A **systematic modeling approach**:
  - **Define the task** - *what do we need to do?*
  - **Functional decomposition** - *how to decompose the functionality?*

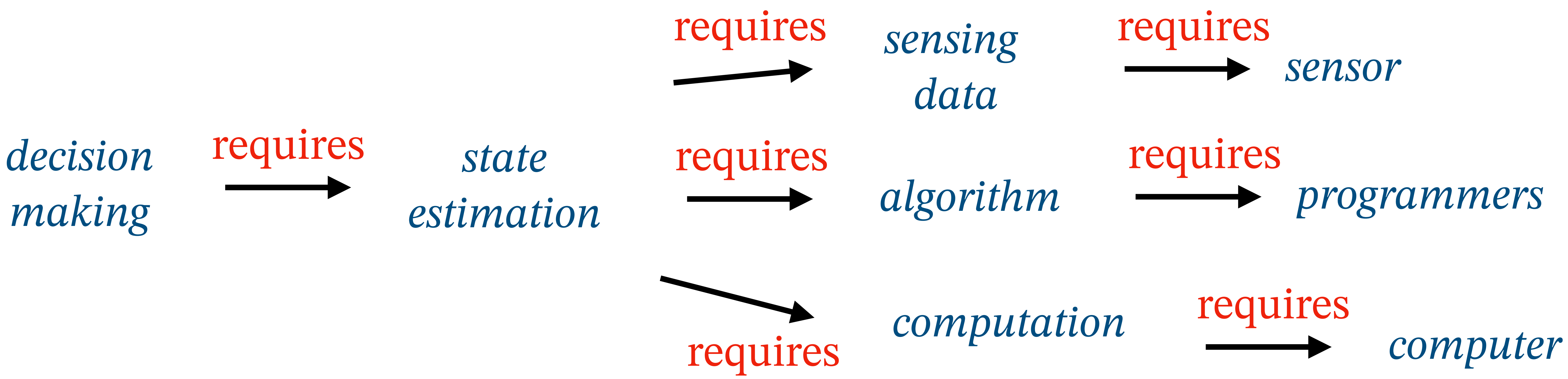


A systematic process for task-driven co-design of complex systems

- A **systematic modeling approach**:
  - **Define the task** - *what do we need to do?*
  - **Functional decomposition** - *how to decompose the functionality?*
  - **Find components** - *decompose until you find components* (hardware and software)



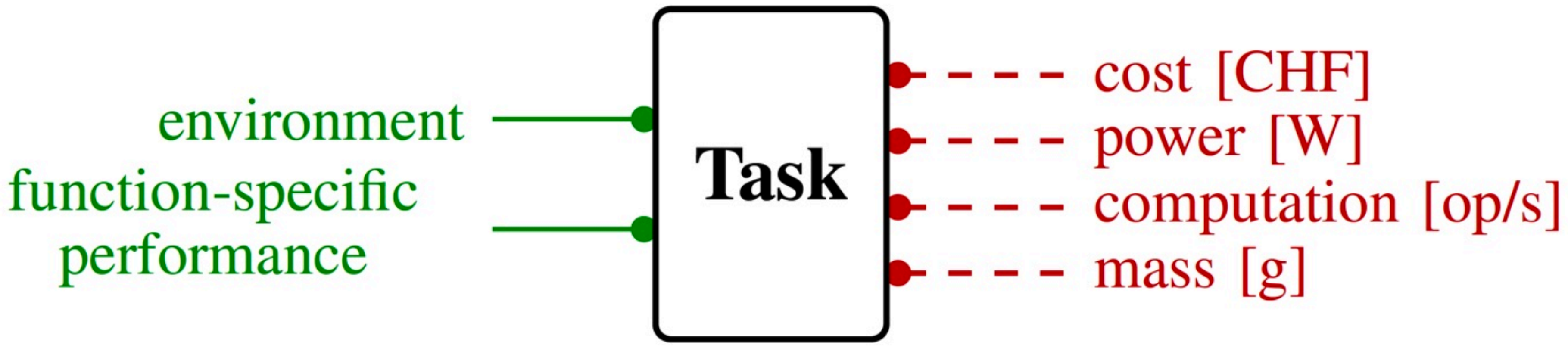
Data/Information flow

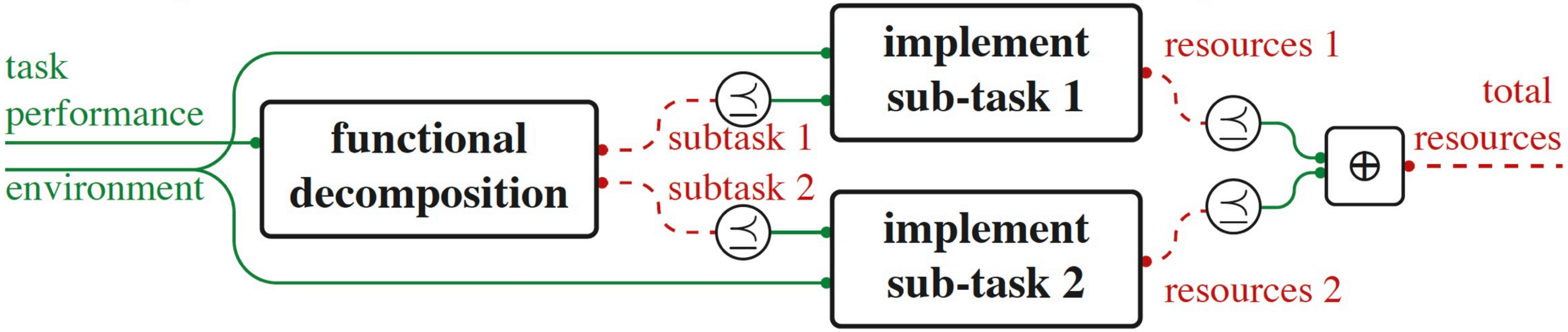


Logical dependencies

A systematic process for task-driven co-design of complex systems

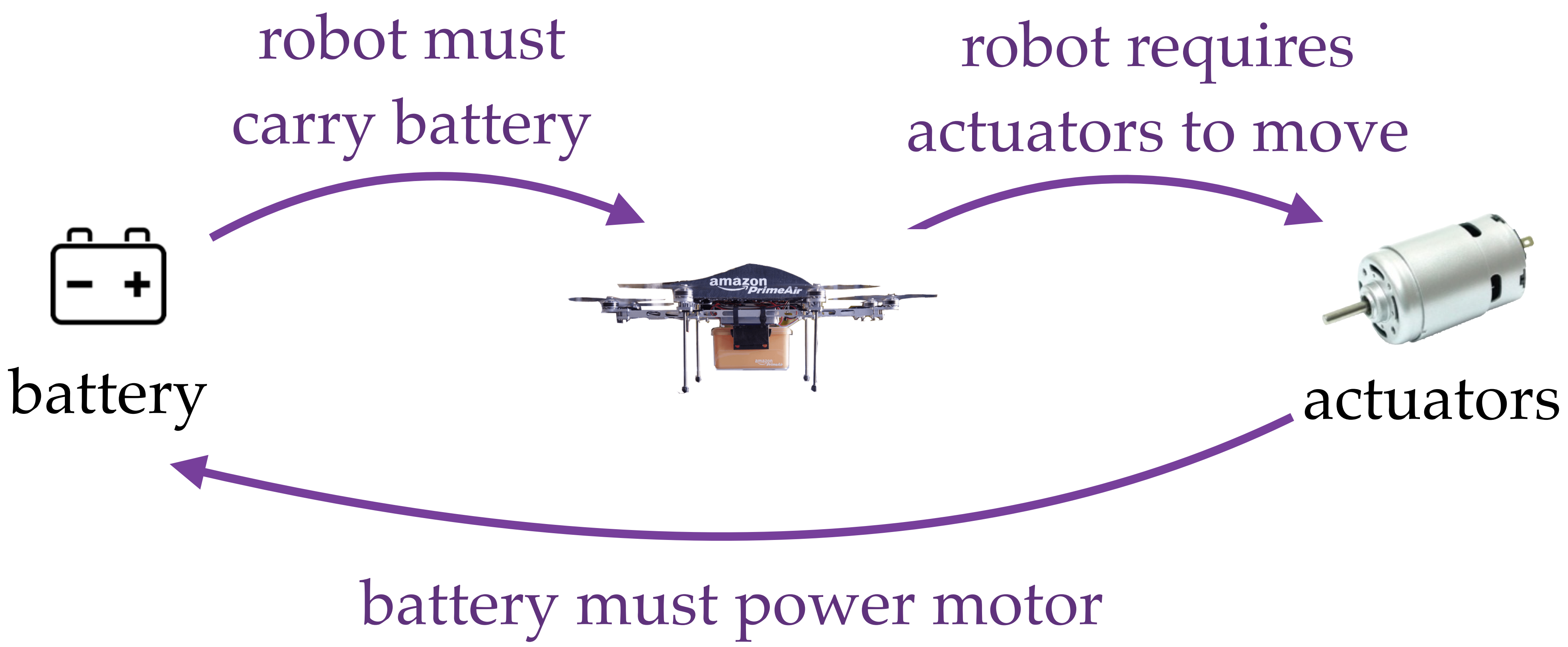
- A **systematic modeling approach**:
  - **Define the task** - *what do we need to do?*
  - **Functional decomposition** - *how to decompose the functionality?*
  - **Find components** - *decompose until you find components* (hardware and software)
  - **Find common resources** and **add** them
    In autonomy, **size**, **cost**, **weight**, **power**, **computation**

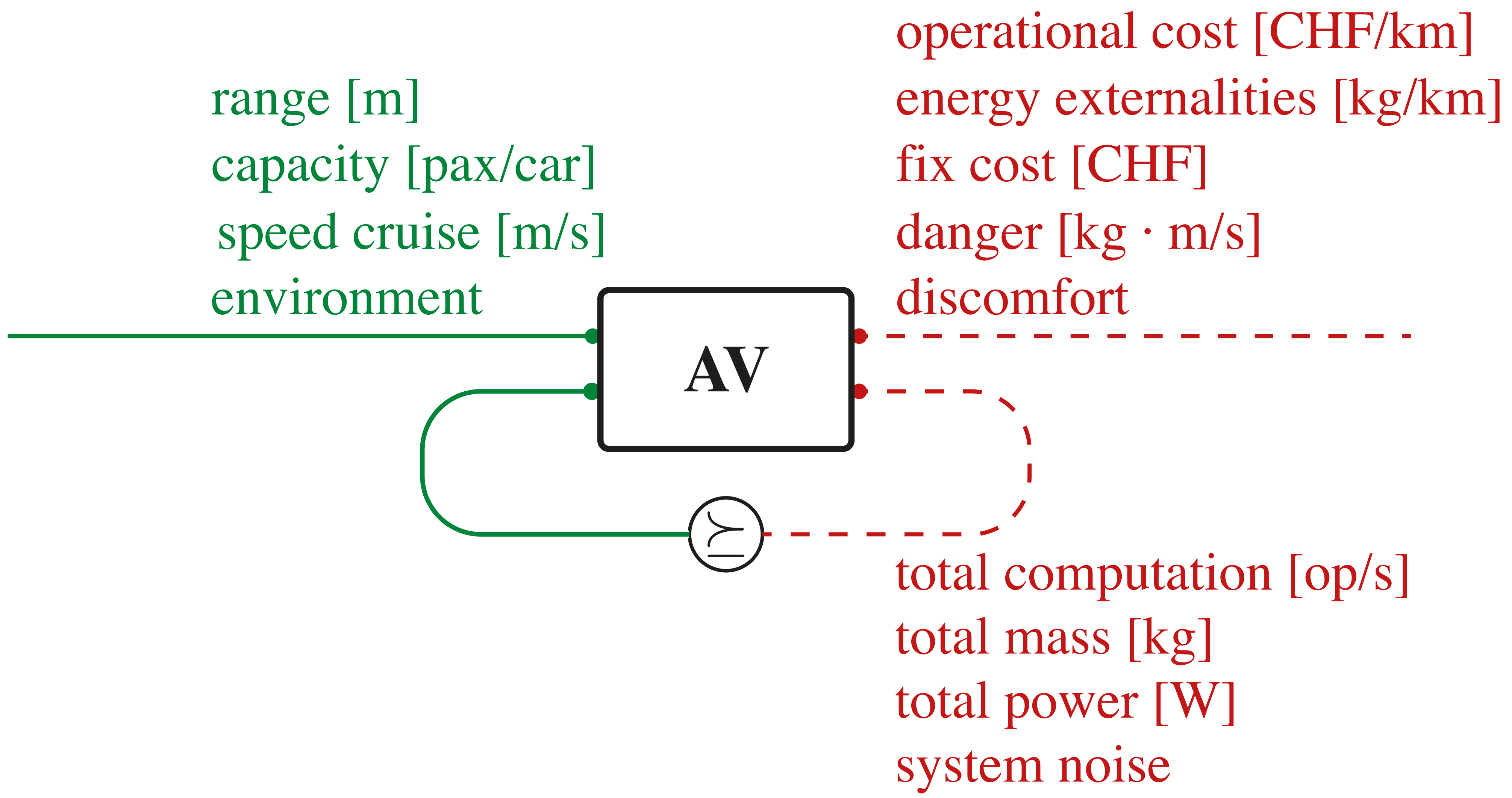


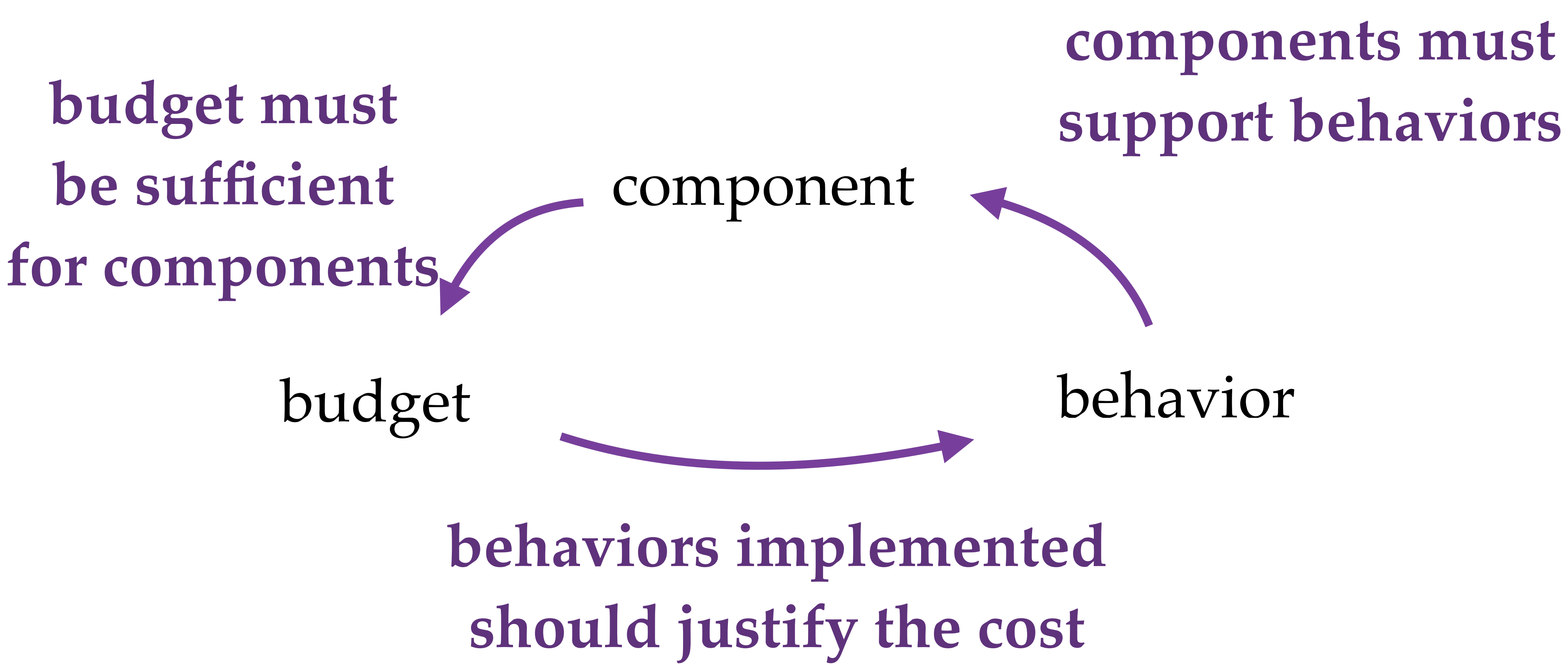


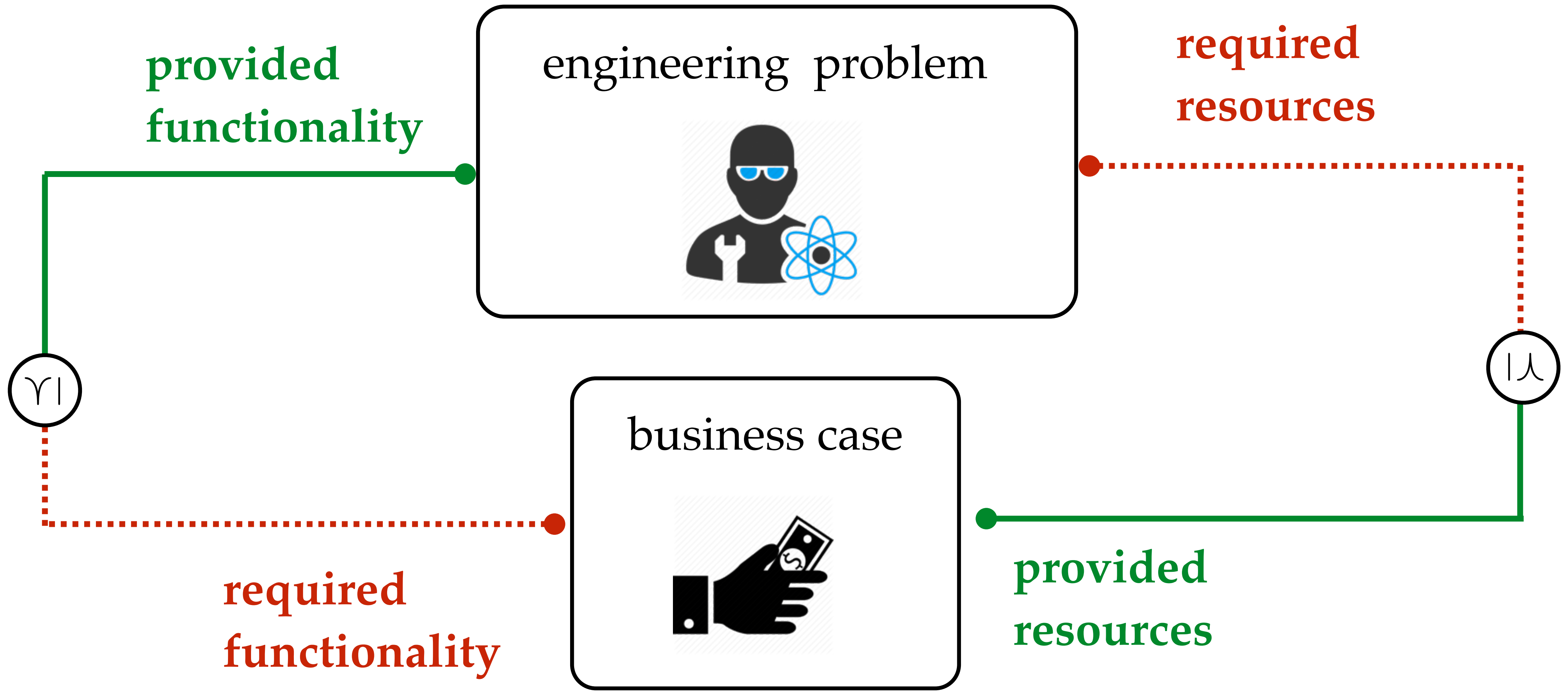
Feedback as the irreducible complexity of system design

- Where is **feedback**? In the **co-design constraints**





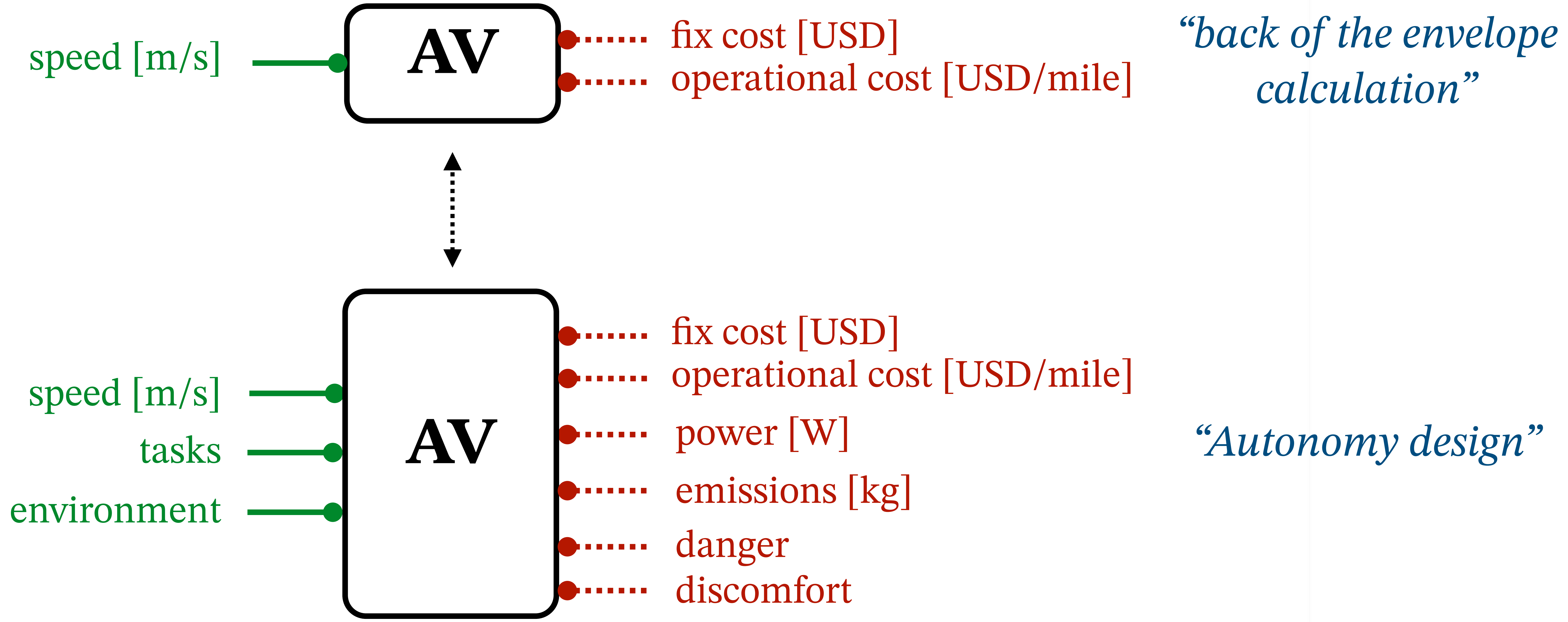




A systematic process for task-driven co-design of complex systems

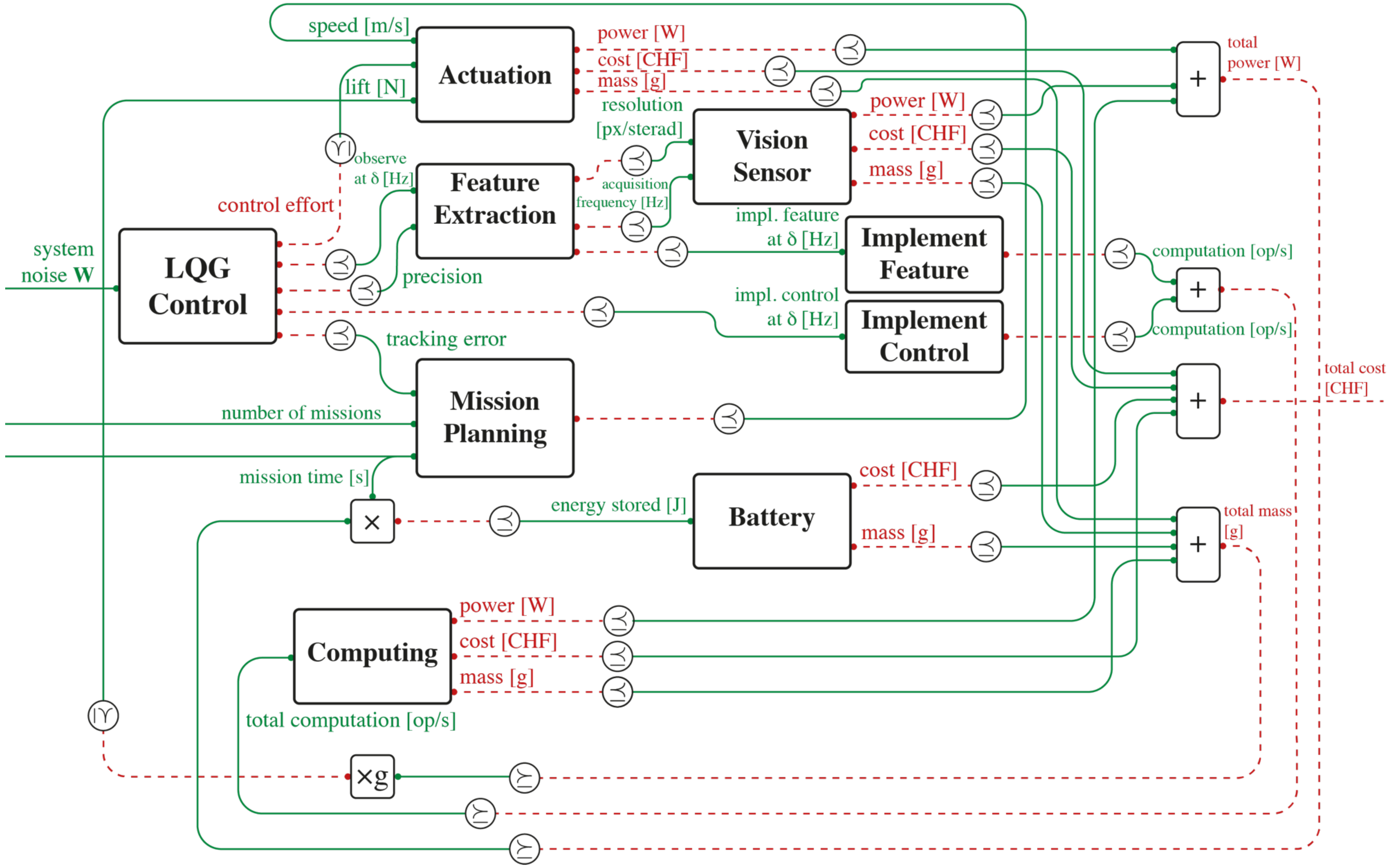
- Actual **implementation**:
  - **Write a skeleton** - *write the structure using the formal language and the* ***logical*** *dependencies.*

Context informs **level of detail**:



A systematic process for task-driven co-design of complex systems

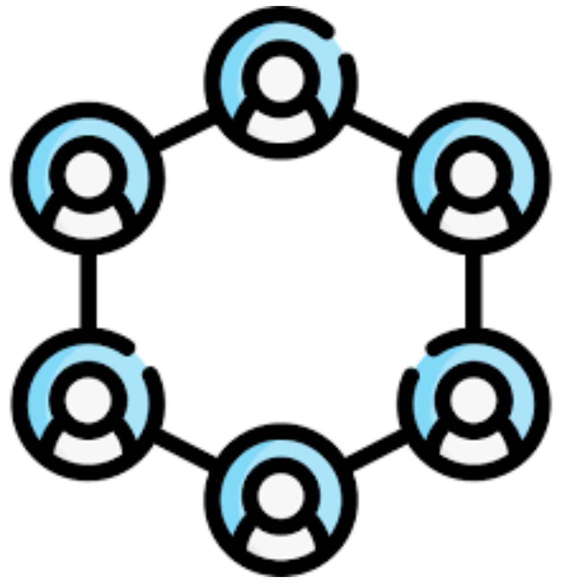
- Actual **implementation**:
  - **Populate the models:**
    catalogues, analytic models, data-driven



Continuous
Collaborative
Intellectually tractable

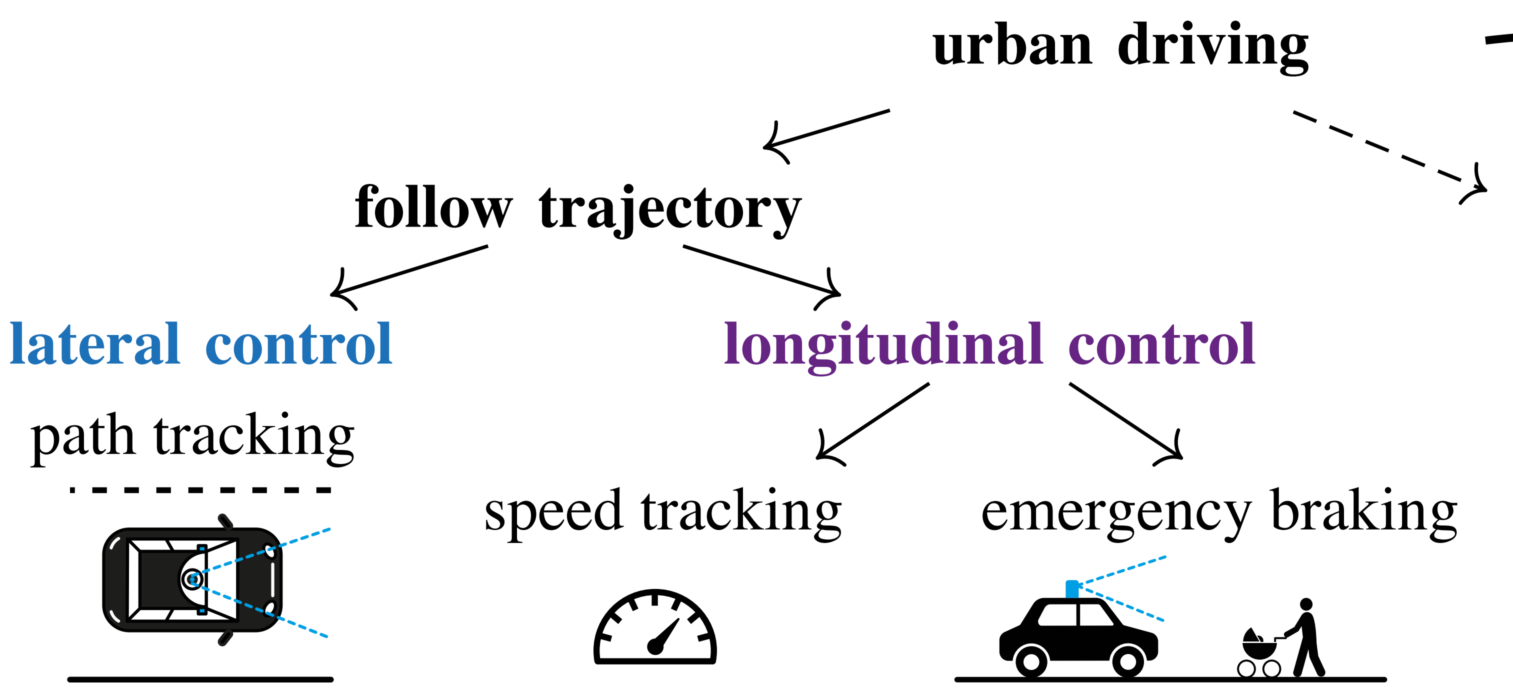
If unsure about a component, easy to embed ***assumptions***

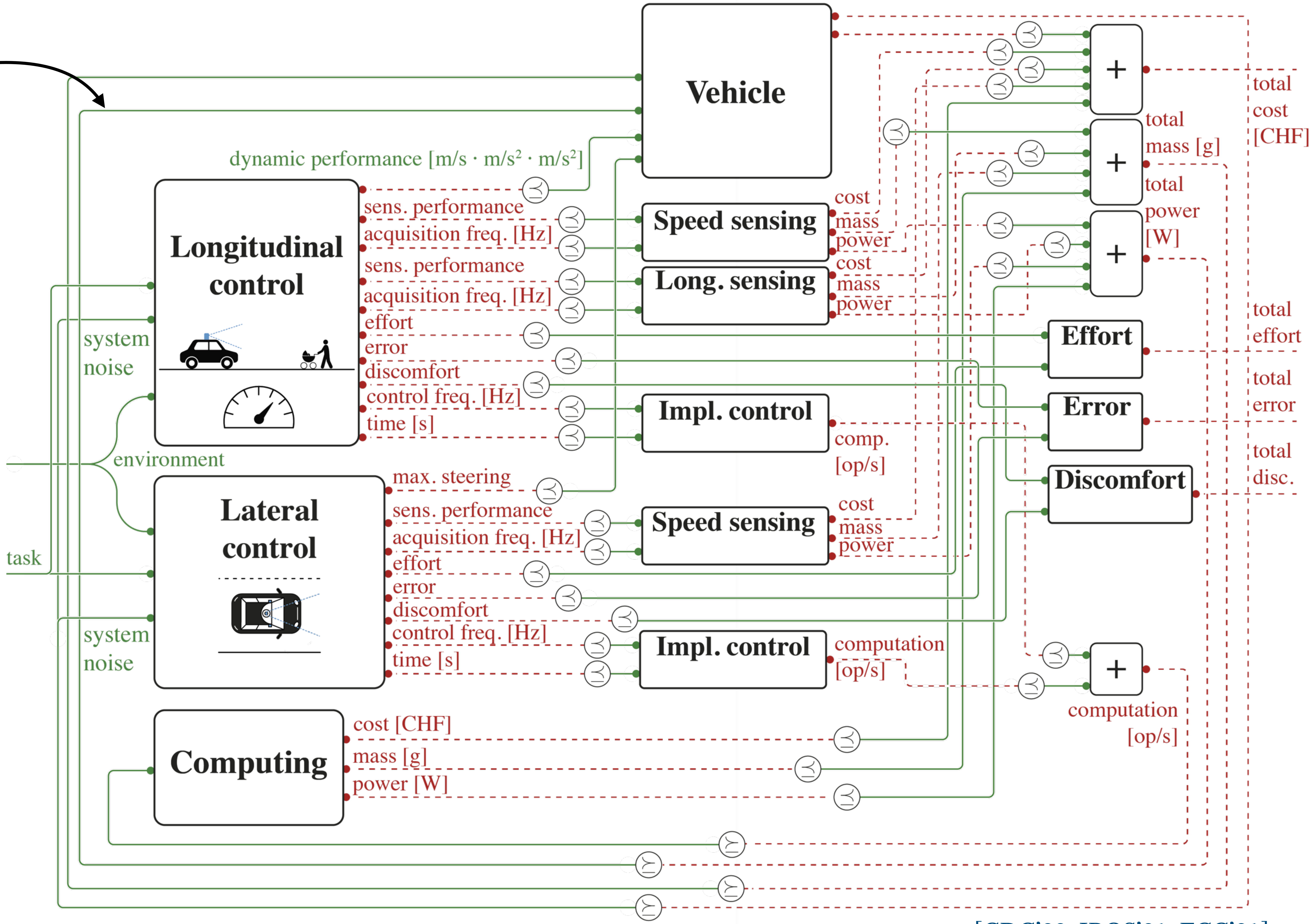
Technologies don't need to exist already - parametric with ***time***



Decentralized - ***humans*** *in the loop*

Task-driven co-design of an autonomous vehicle



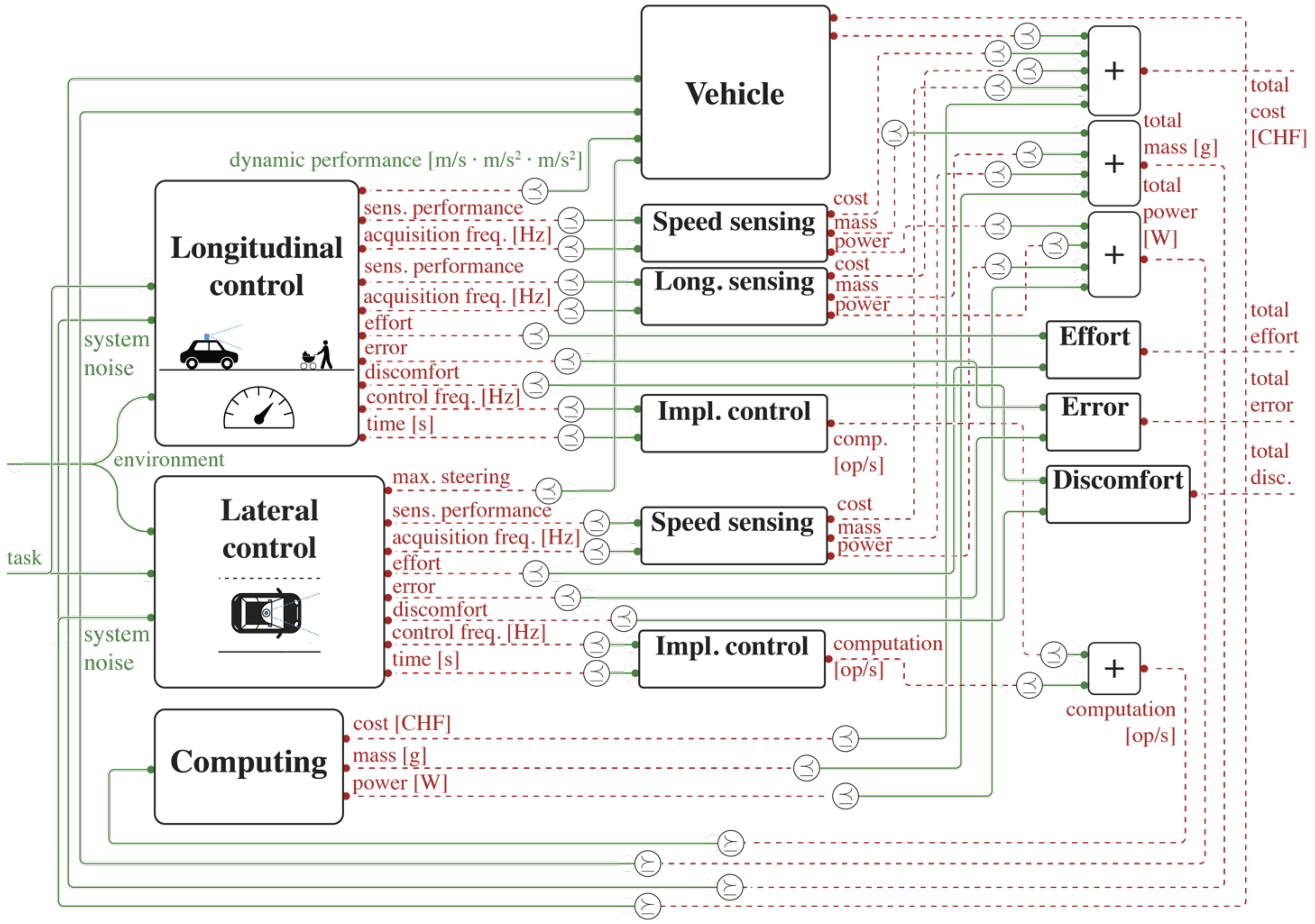


[CDC'22, IROS'21, ECC'21]

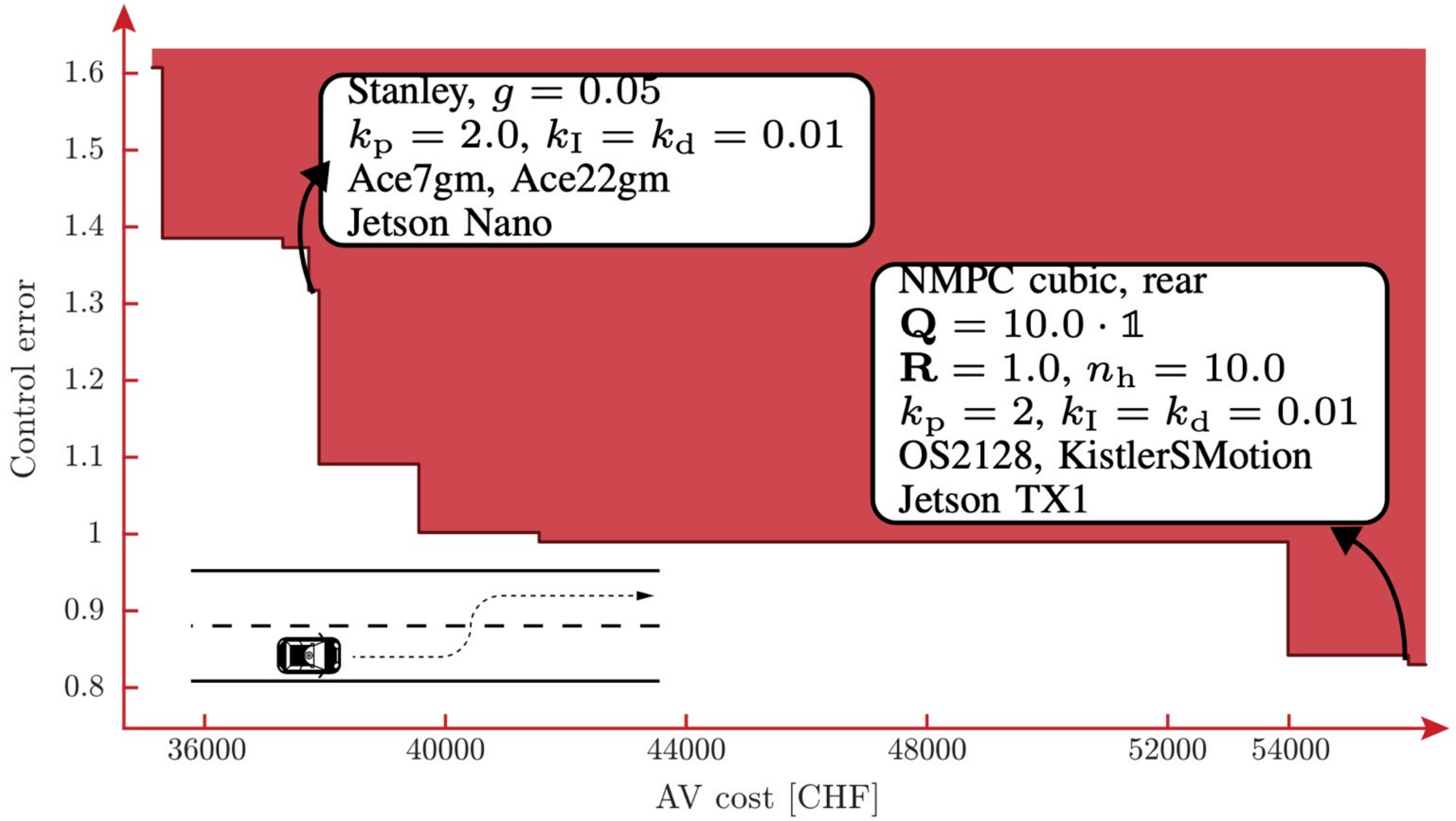
Task-driven co-design of an autonomous vehicle

[CDC'22, IROS'21, ECC'21]

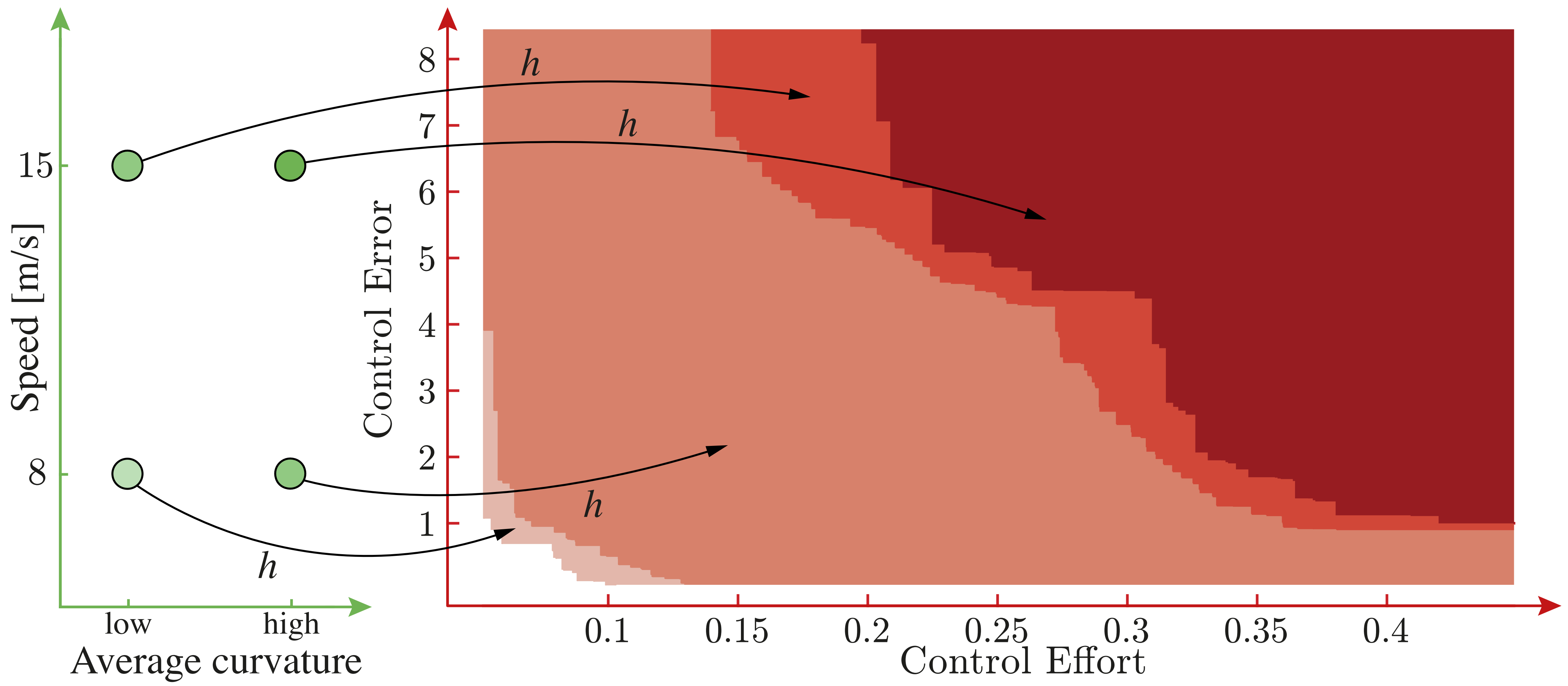
We can find optimal designs, with insights at heterogeneous abstraction levels

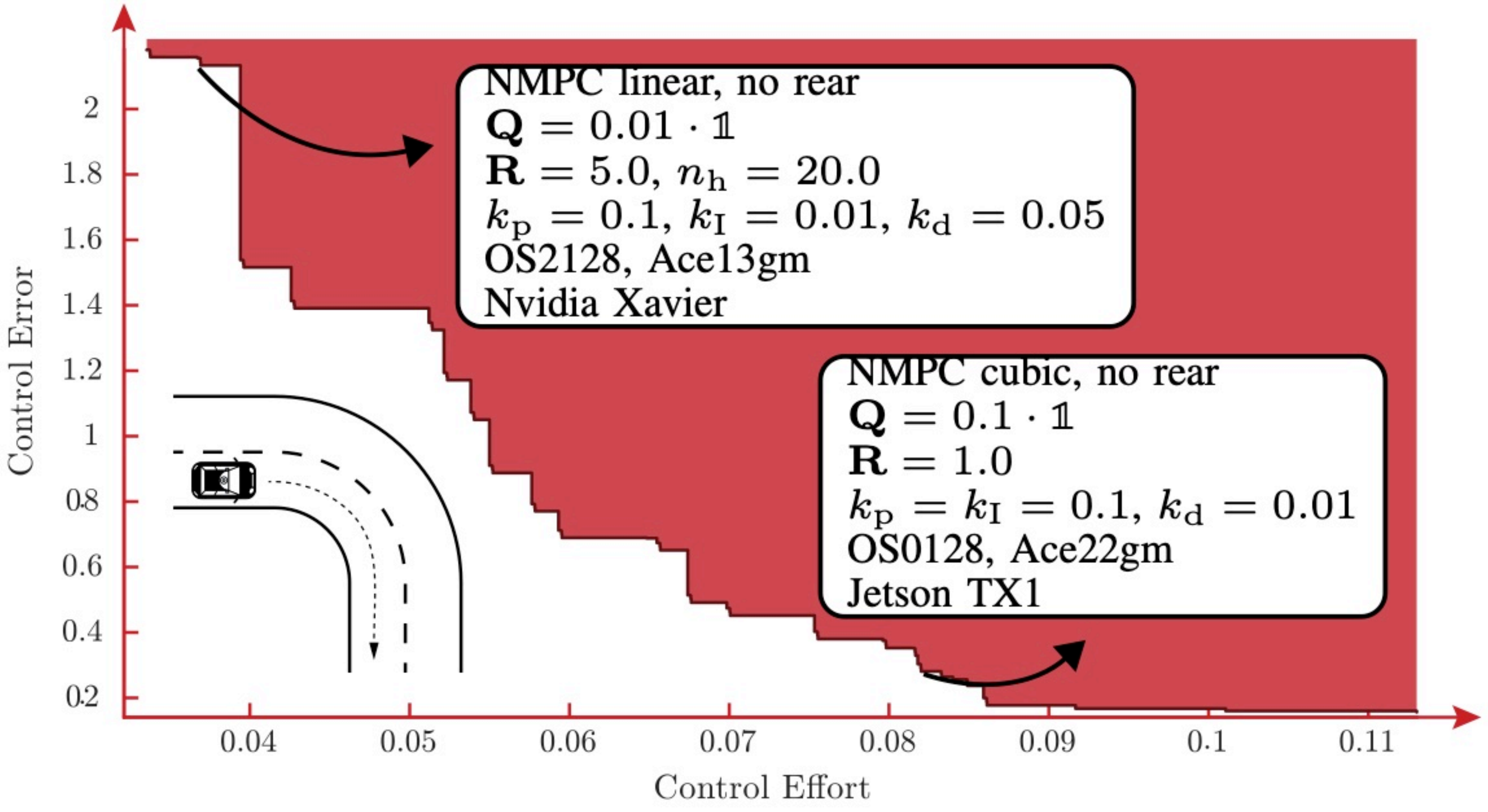


Fix an environment
Fix a task



Monotonicity in task complexity



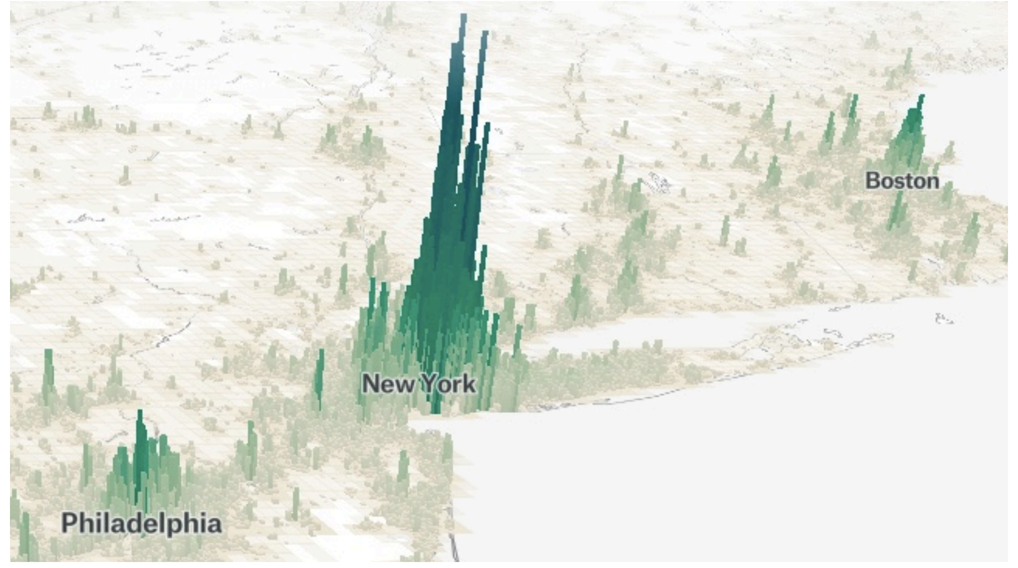


[CDC'22, IROS'21, ECC'21]

Co-design across scales: from autonomy to mobility systems

- Mobility systems are **under pressure**

Travel demand is changing
By 2050, 68% of population in cities



Need for **service design** and **regulations**
Over 1,000% ride-hailing increase in 2012-22

Need to meet **sustainability goals**
Cities cause 60% of GHGs, 30% from mobility

- We look at the problem from the perspective of **municipalities** and **policy makers**

How many vehicles should we allow?

Which infrastructure investments?

How performant?

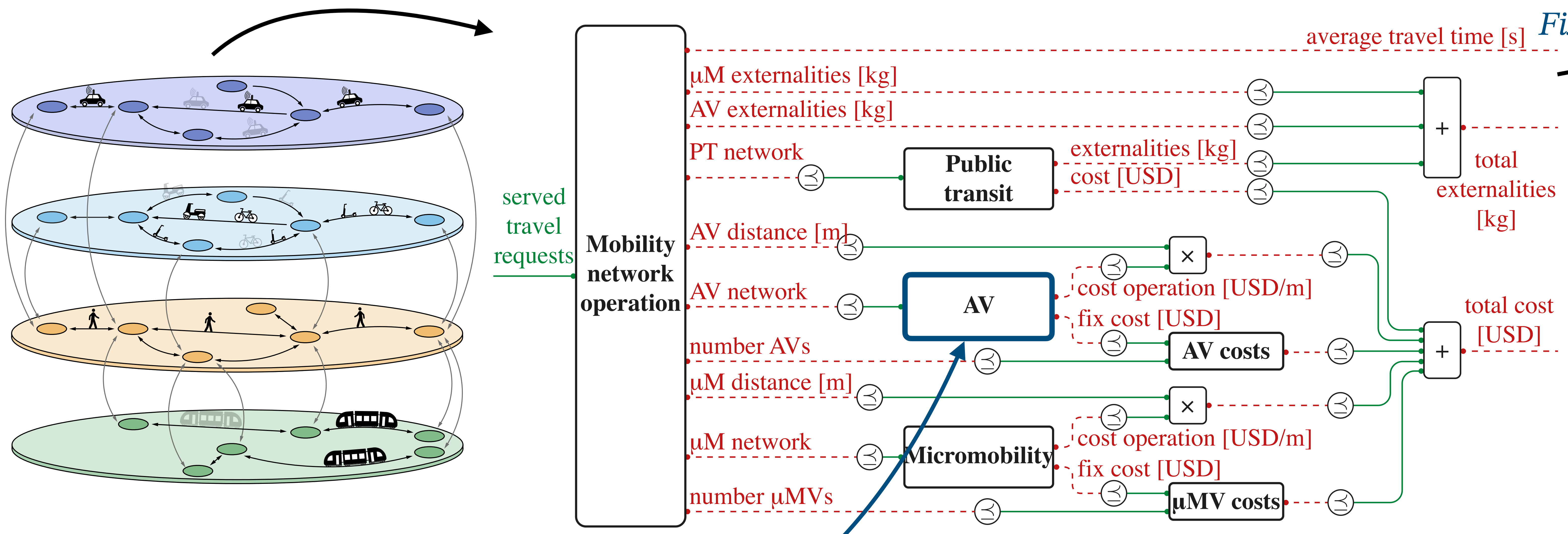
Which services to encourage?

- Need for **demand-driven** co-design of **mobility solutions** and the **intermodal network** they enable

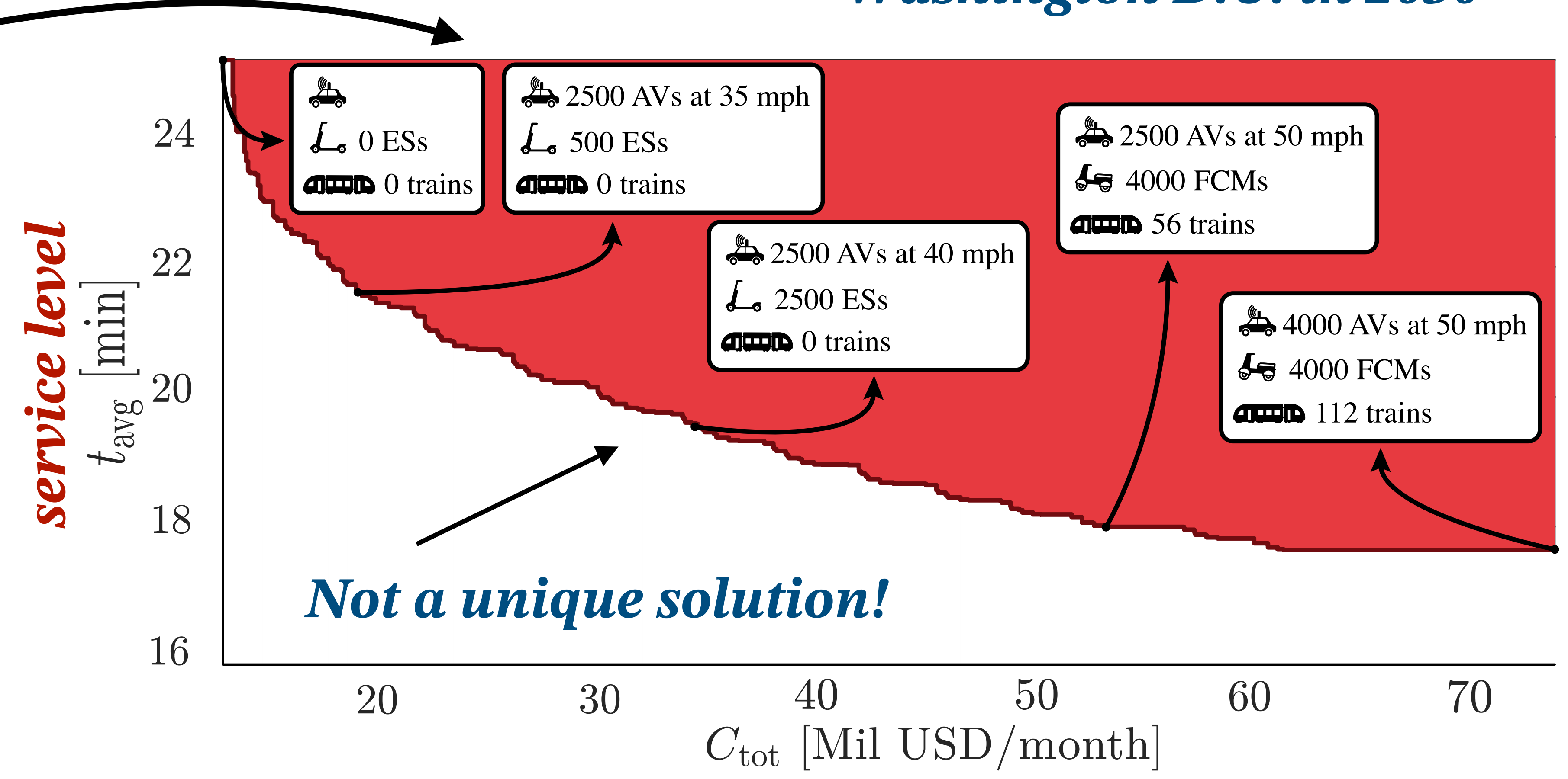
- Several **disciplines** involved (transportation science, autonomy, economics, policy-making)

Co-design to enable user-friendly tools to assess the impact of future mobility solutions

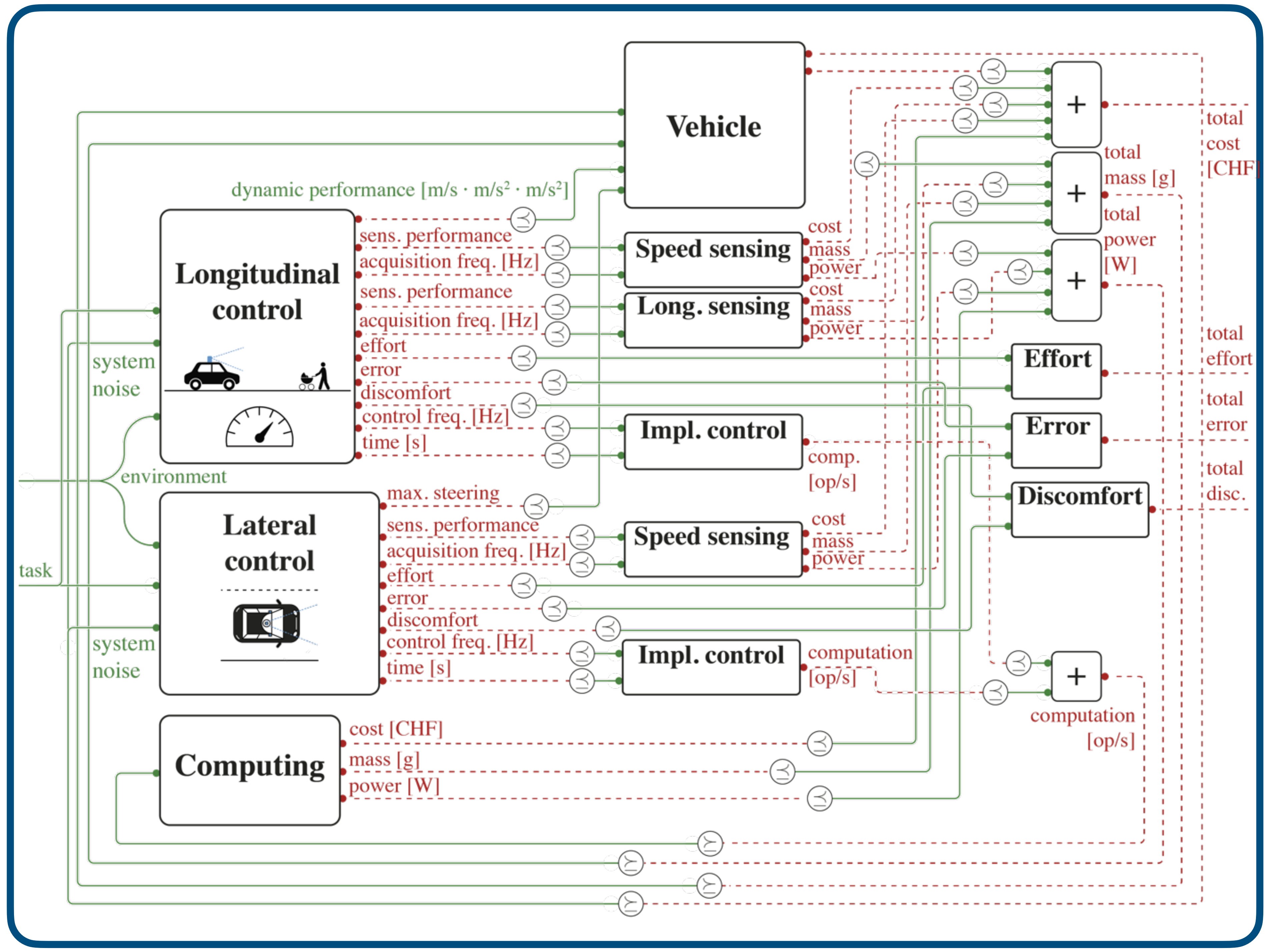
Fix demand, minimize costs



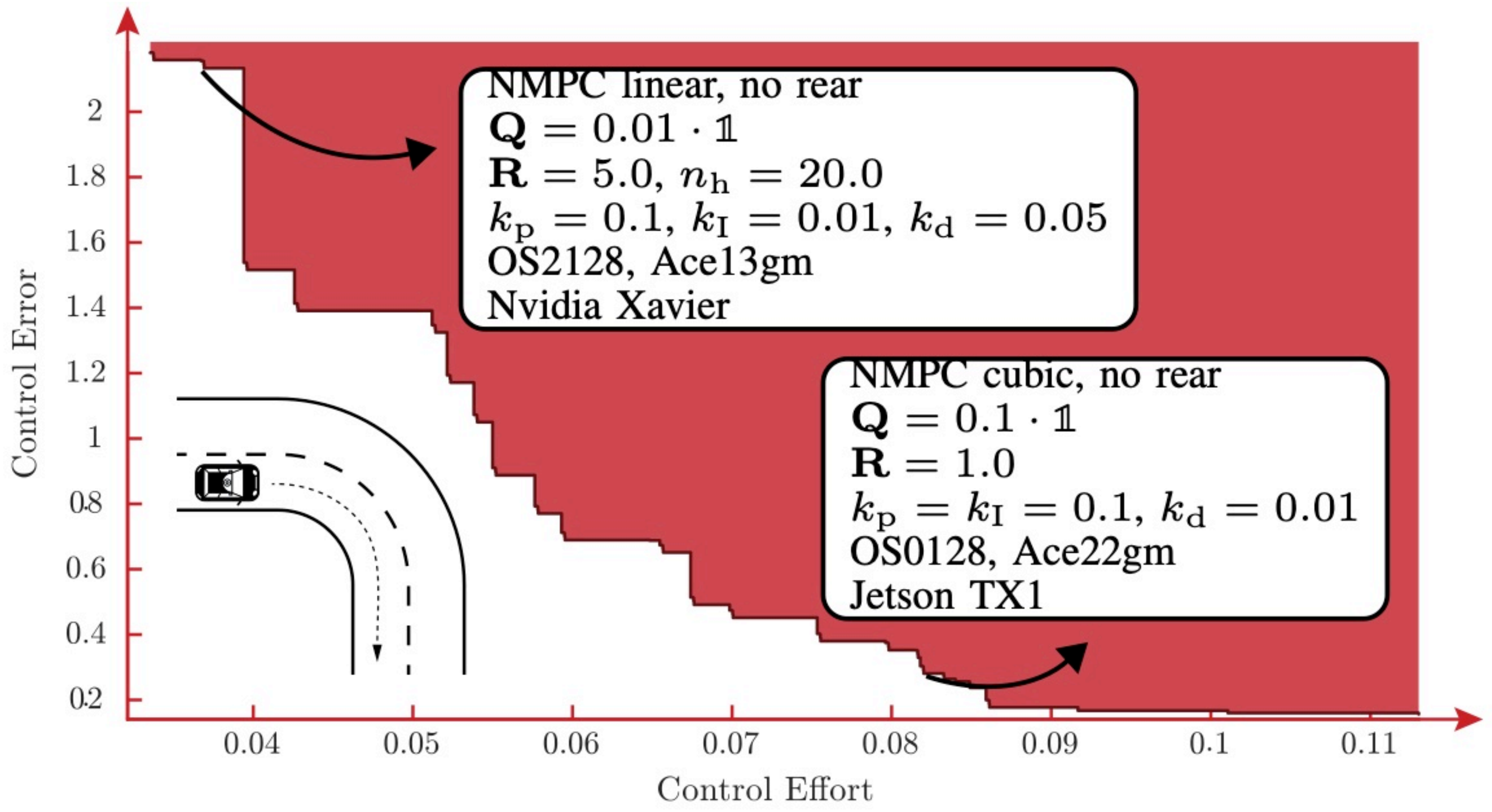
Washington D.C. in 2030



compositionality (vertical composition)



Fix environment, task



Details about software and hardware implementations, in a way that was not possible before

[TNSE'23, CDC'22, IROS'21, ECC'21, ITSC'20]

Agenda

- **Motivation**
  - *New challenges of engineering design*
  - *Motivation from autonomy and mobility*
  - *Desiderata for co-design*
- **Monotone Co-Design**
  - *Modeling design problems*
  - *Examples across domains*
  - *Design queries and optimization*
  - *From autonomy to mobility systems*
- **Strategic interactions**
  - *Game theory to deal with strategic interactions*
  - *Partial order games*
- **Outlook on future research**

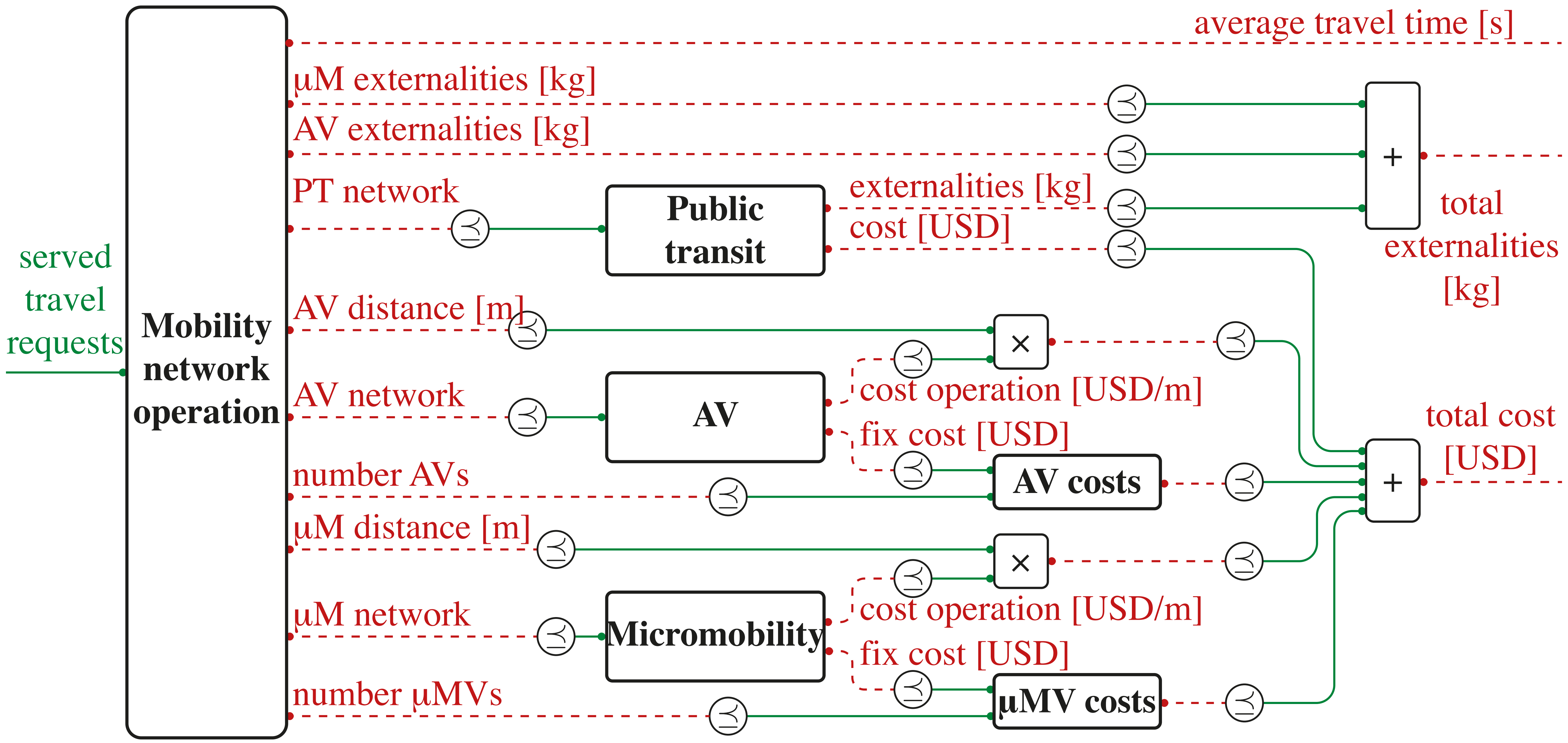
Website containing all papers and more pointers:
https://gioele.science

Complexity when designing complex systems

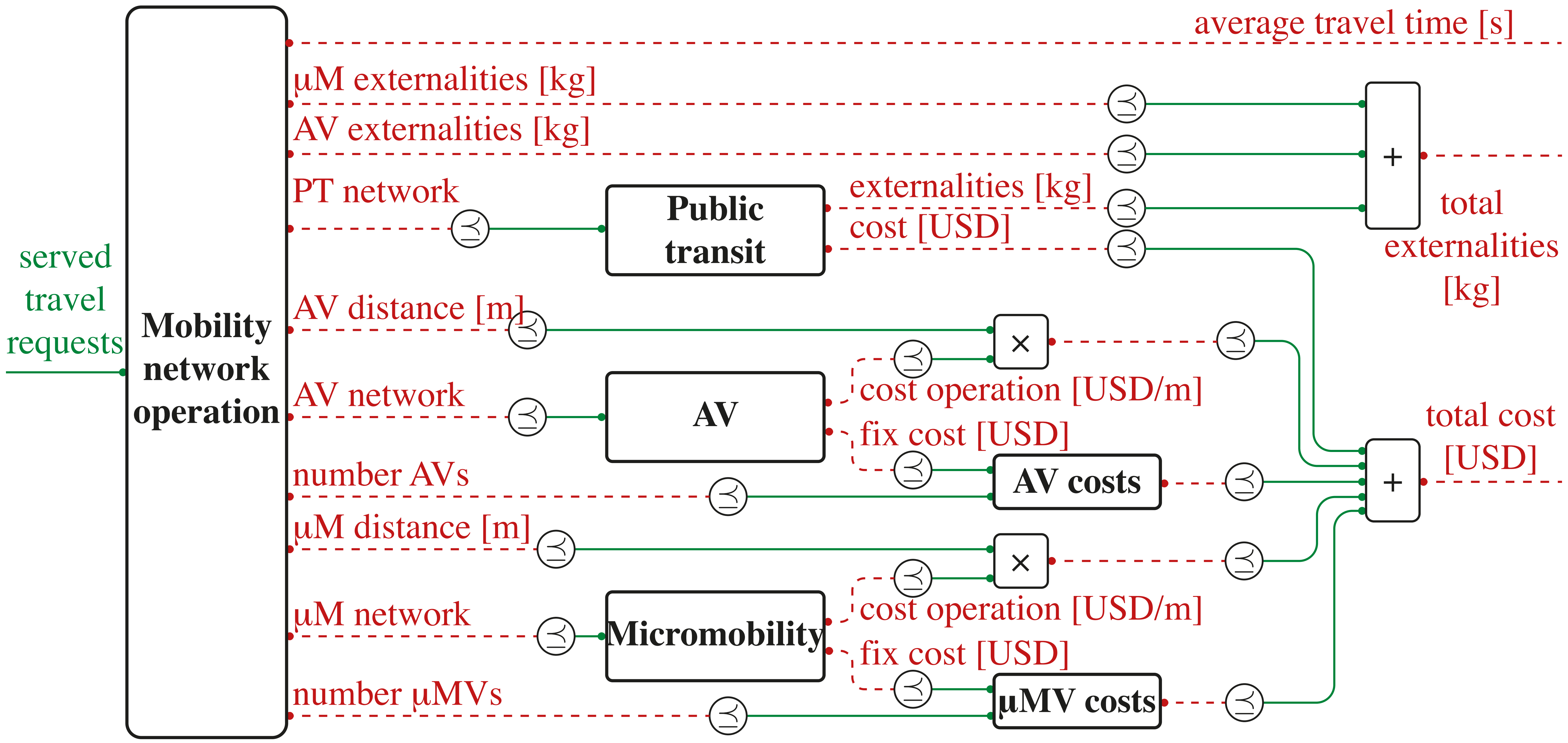
Large systems
- Many components
- Heterogeneous natures
- Multiple objectives

Strategic interactions
- Many agents
- Heterogeneous interactions
- Conflicts/collaborations

Explicitly accounting for strategic interactions: towards co-design games



Explicitly accounting for strategic interactions: towards co-design games



- Different **design problems** belong to different **stakeholders**
- Game theory: Multi-agent **strategic** decision making
  Allows one to **model interactions**
- The notion of **optimal designs** extends to **equilibria of designs**
- Towards a theory of **co-design games**

- Two **milestones** towards **co-design games**:

Co-design features **rich cost structures** (posets):

- **"Posetal Games"** (games with posetal preferences)

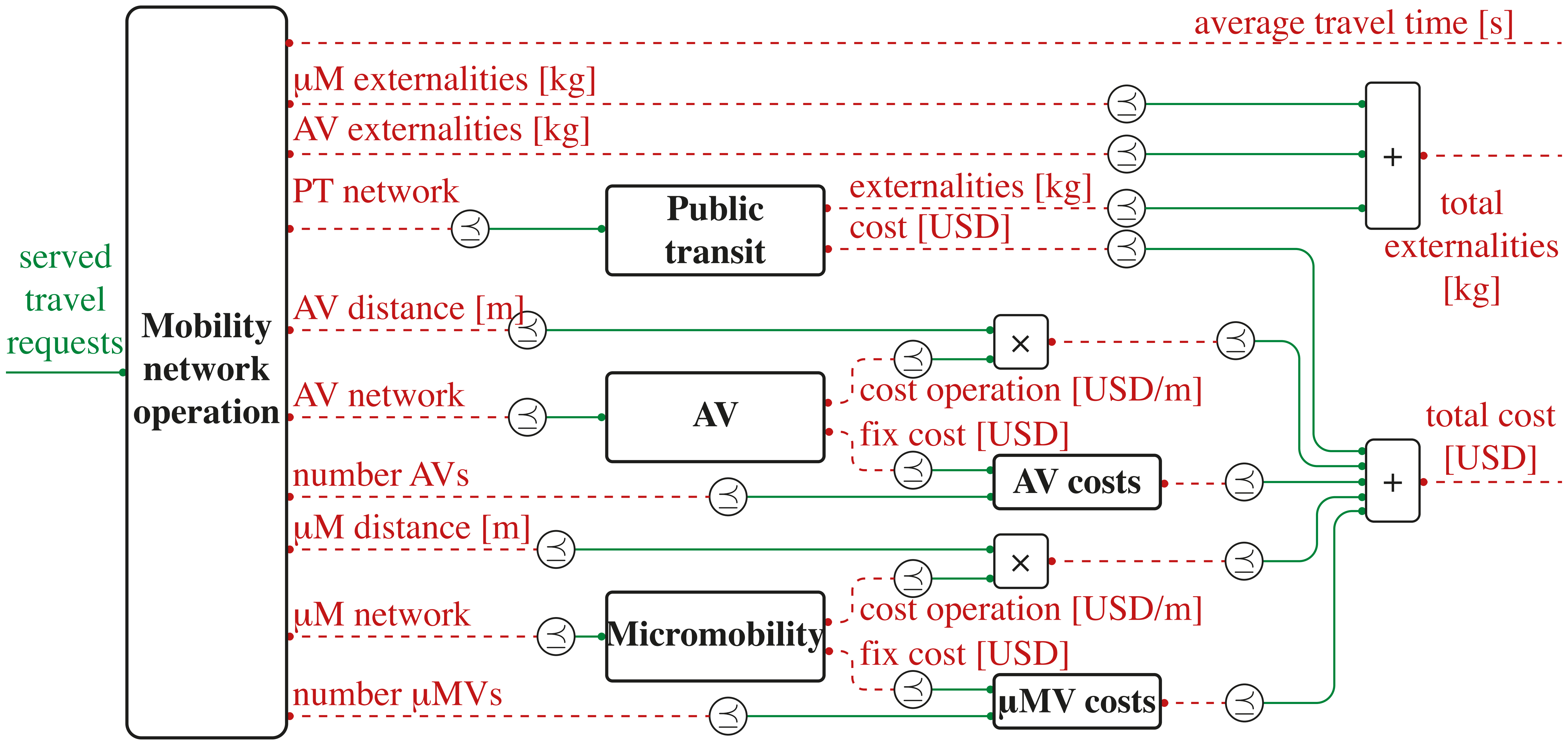
[RA-L' 22]

Interactions are naturally **hierarchical**:

- **Mobility games** via Stackelberg

[ITSC'21 (***Best Paper Award***), ITSC'23]

Explicitly accounting for strategic interactions: towards co-design games



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  [RA-L' 22]

  Interactions are naturally **hierarchical**:
  - **Mobility games** via Stackelberg

  Next time!

  [ITSC'21 (***Best Paper Award***), ITSC'23]

Behavior requirements for robots are numerous, vague, and conflicting

Ethics

Safety

Liability

Compliance to traffic rules

Extensive & diverse
written by humans for humans

Function

"Does your car have any idea why my car pulled it over?"

Courtesy

Culture

Example: Boston left

Comfort

Safety for human-driven vehicles

- Safety (i.e., prevention of *unreasonable risks of driving*) is typically ensured by a mix of:
  - **Certification** of vehicles and drivers
  - **Rules** of the road
  - **Enforcement** by authorities and legal system

- Typically, **rules** rely on fundamental **axioms**, which require **interpretation**

Fundamental norm in Switzerland:
All road users must behave in such a way not to pose an obstacle or a danger to other road users

- No clear specification of **safety**
- It is **legal** to break the law to ensure **safety**

Things that do not work well for AV behavior specification

- **Hard constraints**
  - What do you do with **infeasibility**?
  - Whenever you consider other actors, hard to find **guarantees**

- **Case analysis, finite state machines, …**
  - "IF statements kill people"

- **Just relax!**

$$J = \alpha J_1 + \beta J_2 + \gamma J_3 + \ldots$$

  - Hard to re-tune, prone to **overfitting**
  - Lack of **transparency**

What should we do instead?

- Throw the ball at **other stakeholders**
- Incorporate our **own beliefs** in our algorithms

- Create **transparent** systems
- Create **customizable** systems
- **Explain** issues to the public
- **Engage** with **stakeholders** of the problem (e.g., regulators, liability companies, etc.)

Minimum violation planning

- Assume that constraints will be violated, and find the alternative that ***least*** violates them
- Define **rules** as a **total order** over realizations
- **Order rules** according to priority
- This is practical:
  - Allows **modular definition of behavior**
  - **Easy to predict** what the car will do
  - **Easy to understand** why the car did something
  - One can introduce **tolerances**

Taxi	**Race car**
Safety	*Compliance*
↑	↑
Compliance	*Performance*
↑	↑
Comfort	*Safety*
↑	↑
Performance	*Comfort*

What if rules are incomparable, or indifferent?

See seminal work by Tumova, Karaman, Frazzoli

We capture the richness of robot behavior requirements via partial orders

- We can use **pre-orders over rules** to express preferences

"Rule A is more important than rule B"

A
|
B

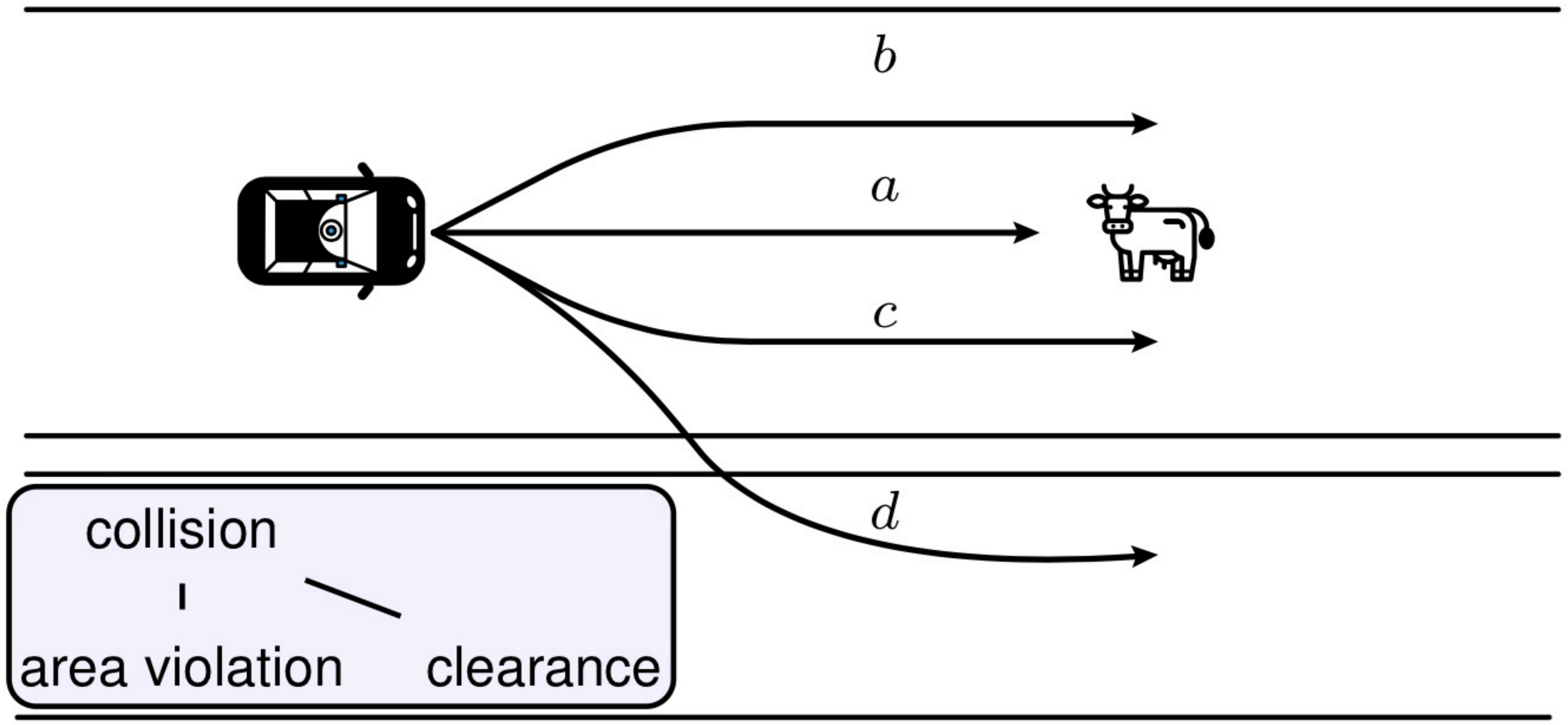
"Rule A and B are not comparable"

A *B*

"Rule A and B are indifferent"

A ⟷ *B*

- Pre-order over rules induces pre-order over outcomes



b and *c* are **indifferent**

b, c, d are **preferred** over *a*

b, c are **incomparable** with *d*

Minimum violation planning using partial orders, unbridled creativity and good taste

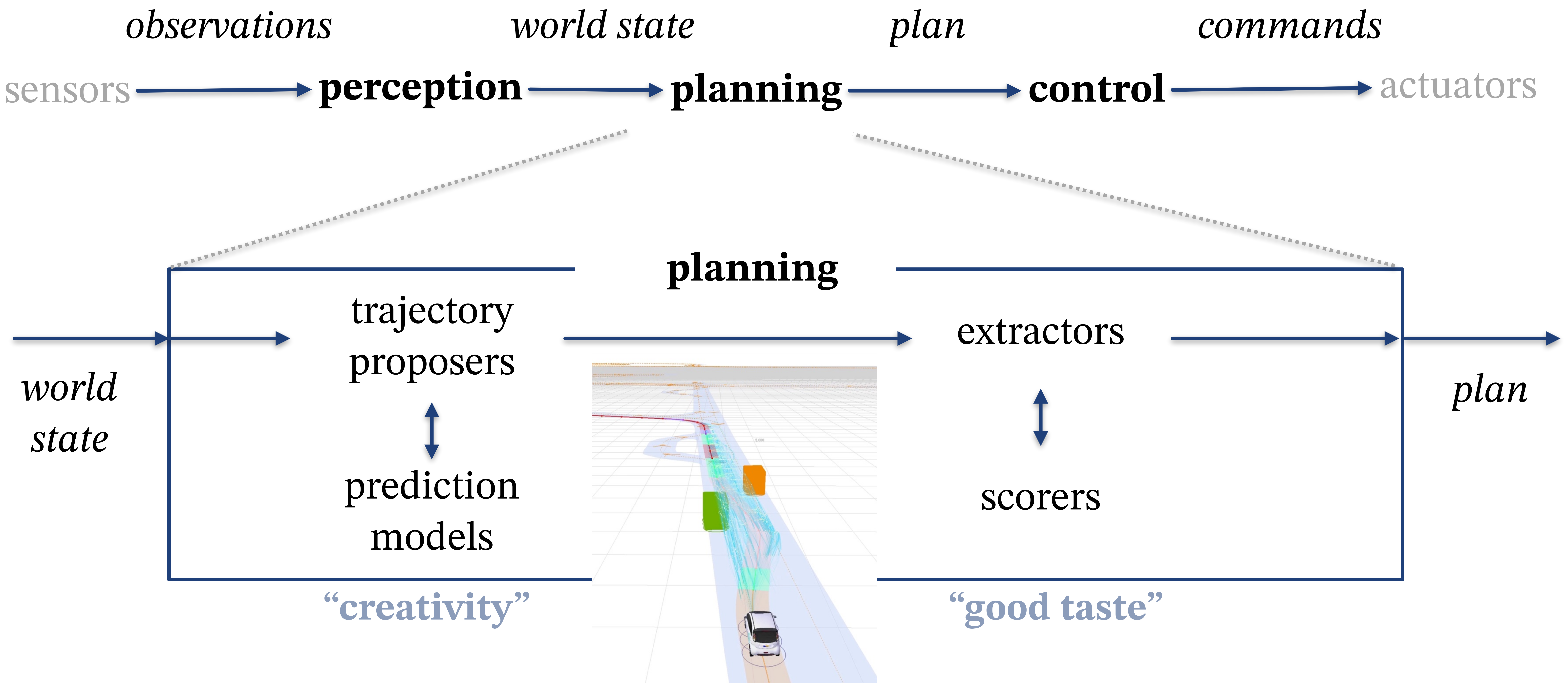
“The way to get good ideas is to get lots of ideas, and throw the bad ones away.” — Linus Pauling

creativity

good taste

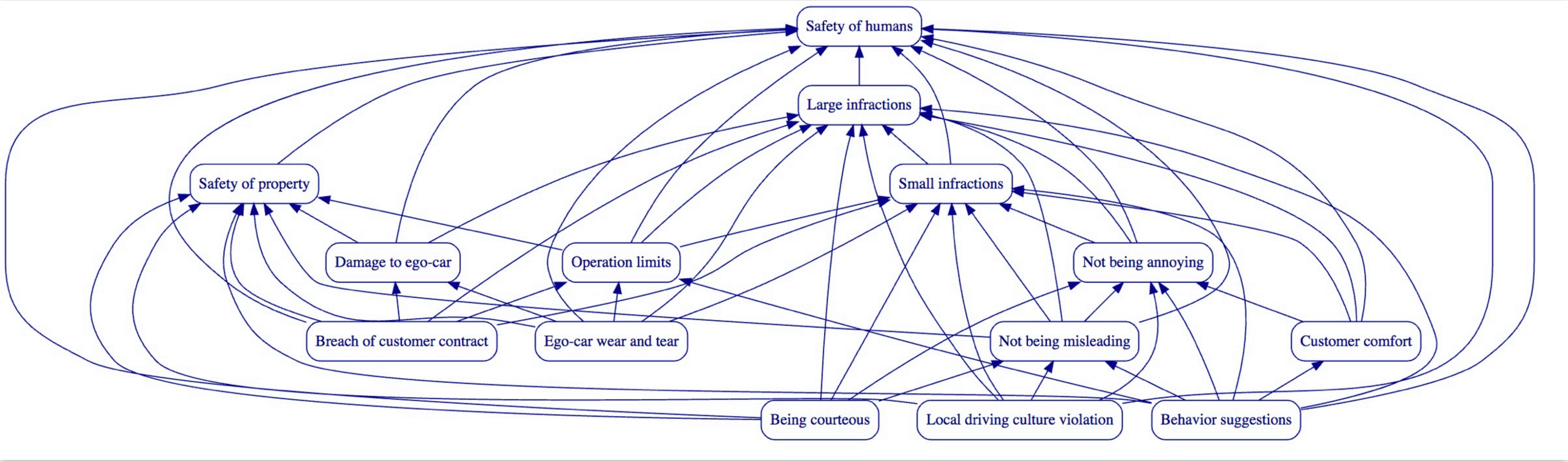
See Rulebooks work by Censi, Frazzoli, and others

Minimum violation planning using partial orders, unbridled creativity and good taste



Defining and ordering rule groups for realistic scenarios

- Estimate: urban driving requires ~200 **rules**, ~20 **rule groups**



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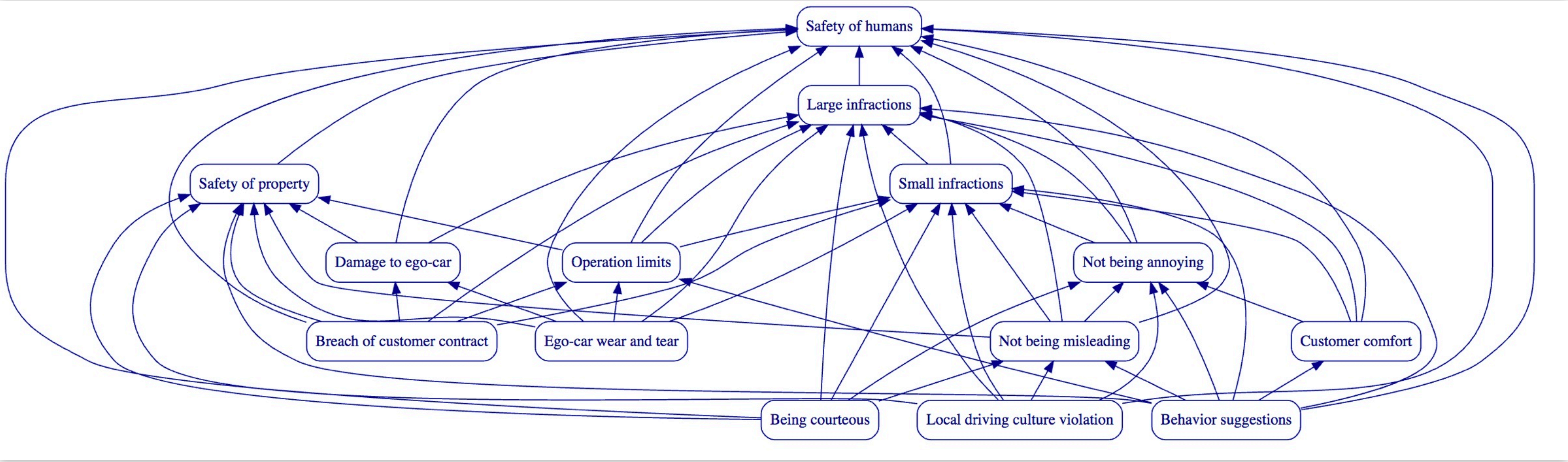


All of this is considering ego agents…
How do these specifications work with multiple, interacting agents?

Posetal Games to deal with highly interactive multi-objective nature of decisions

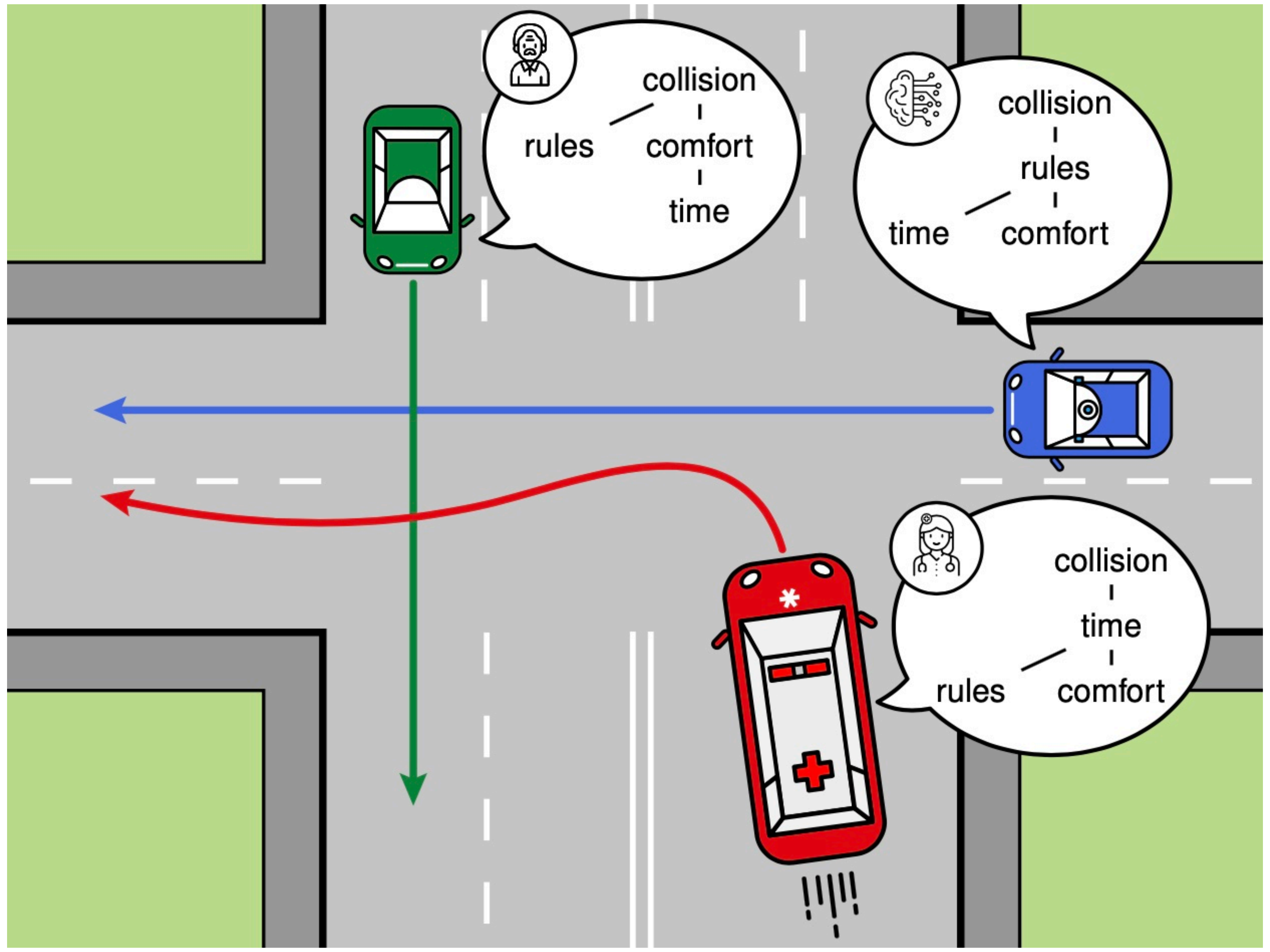
- **Games** in short:
  - Each **player** has a **scalar utility function**
  - Based on **preferences**, players select an **action** from decision space
  - Given joint **action profile** of players, we obtain a **game outcome** for each player via a *deterministic* **metric function**
  - **Equilibria** are joint action profiles from which **no player** has **interest to deviate**

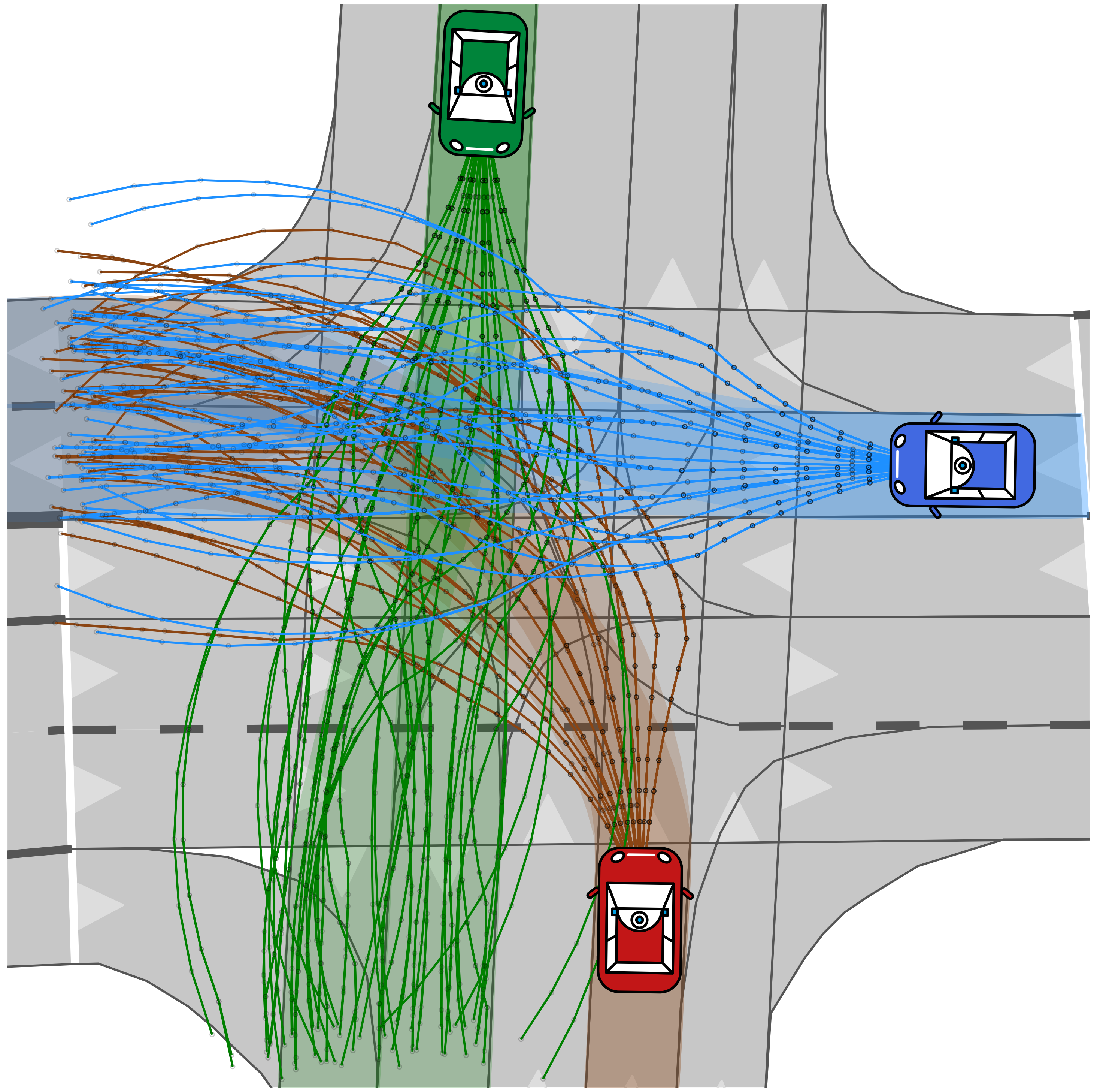
Related work: V.V. Rozen'18, S. Le Roux'08

Posetal Games to deal with highly interactive multi-objective nature of decisions

- **Posetal games** in short:
  - Each **player** has a ~~scalar utility function~~ *partially ordered* **preference** over a set of metrics (scores, costs)
  - Based on **preferences**, players select an **action** from decision space
  - Given joint **action profile** of players, we obtain a **game outcome** for each player via a *deterministic* **metric function**
  - **Equilibria** are joint action profiles from which **no player** has **interest to deviate**

Technical results instantiated in trajectory driving games for urban scenarios

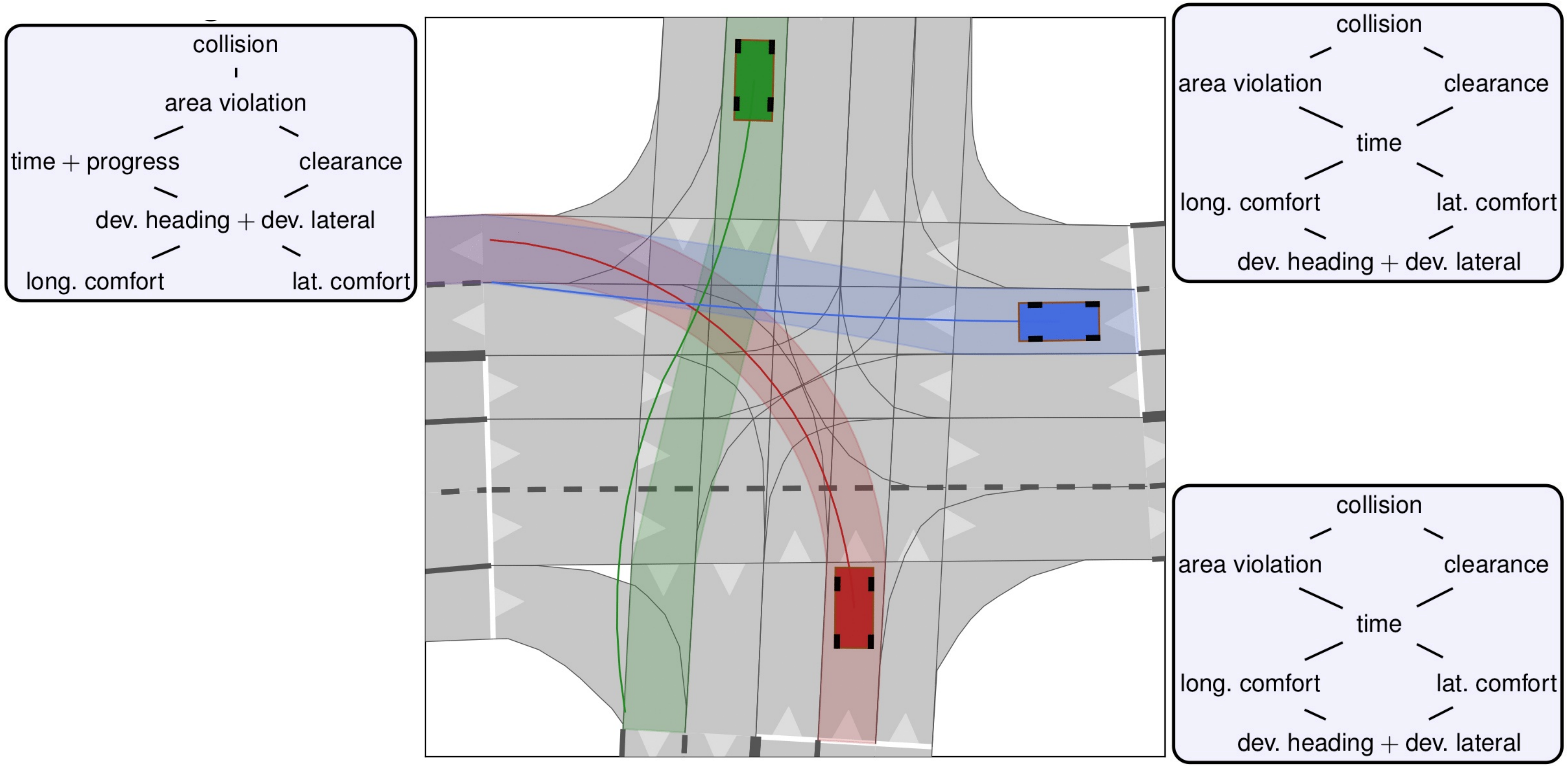




[RA-L' 22]

Posetal Games to deal with highly interactive multi-objective nature of decisions

- Posetal games **extend standard notions in game theory**, and
  - Provide **sufficient** conditions for the existence of **Nash equilibria** (via **potential games**)
  - Characterize **efficiency** of **admissible equilibria**
  - Design a **formal, systematic** way to leverage **preference refinement** (e.g., via *estimation*) to **refine equilibria**



Agenda

- **Motivation**
  - *New challenges of engineering design*
  - *Motivation from autonomy and mobility*
  - *Desiderata for co-design*
- **Monotone Co-Design**
  - *Modeling design problems*
  - *Examples across domains*
  - *Design queries and optimization*
  - *From autonomy to mobility systems*
- **Strategic interactions**
  - *Game theory to deal with strategic interactions*
  - *Partial order games*
- **Outlook on future research**

Modeling & Algorithmic Foundations

Societal Applications

User-friendly Tools

Website containing all papers and more pointers:
https://gioele.science

My lab will be building the next generation tools for systems design optimization

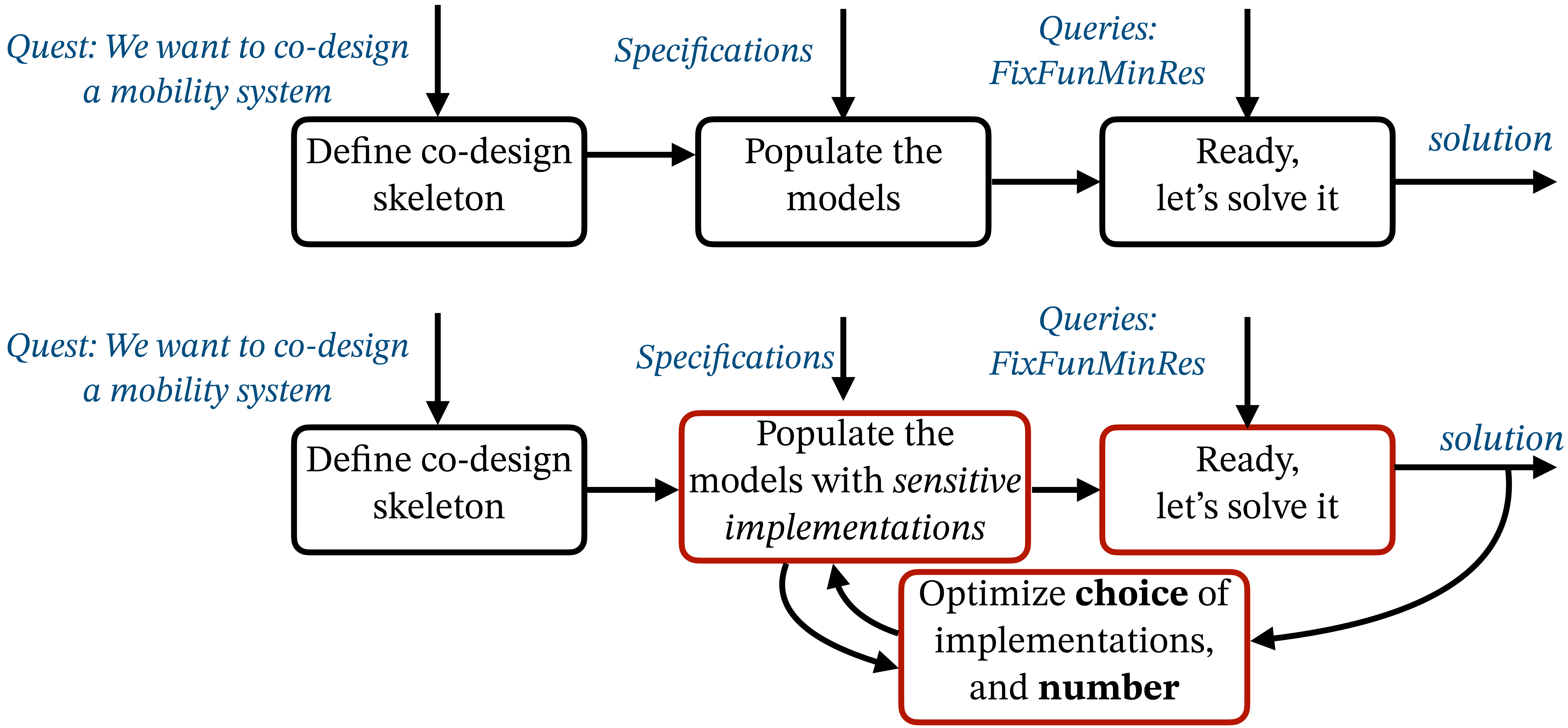
Modeling and Algorithmic Foundations

Leveraging **optimization**, **control theory, game theory, domain theory**, and **applied category theory**:

- Extend and improve current **modeling** & **solution algorithms** for **multi-objective** design optimization
- Promote **interdisciplinarity** by bridging the gap between **standard optimization** and **co-design**
- Explicitly account for **strategic interactions** of stakeholders, developing a theory of **co-design games**

Example:
Computation-aware iterative solution algorithms

*How to best **change** the **approximation** of each **model** **adaptively** and **dynamically**?*



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How to leverage **negative information** for design?

Categorification of Negative Information using Enrichment

Andrea Censi Emilio Frazzoli Jonathan Lorand Gioele Zardini

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Department of Mechanical and Process Engineering
ETH Zurich, Switzerland
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In many engineering applications it is useful to reason about "negative information". For example, in planning problems, providing an optimal solution is the same as giving a feasible solution (the "positive" information) together with a proof of the fact that there cannot be feasible solutions better than the one given (the "negative" information). We model negative information by introducing the concept of "norphisms", as opposed to the positive information of morphisms. A "nategory" is a category that has "nom"-sets in addition to hom-sets, and specifies the interaction between norphisms and morphisms. In particular, we have composition rules of the form morphism + norphism → norphism. Norphisms do not compose by themselves; rather, they use morphisms as catalysts. After providing several applied examples, we connect nategories to enriched categtory theory. Specifically, we prove that categories enriched in de Paiva's dialectica categories **GC**, in the case **C** = **Set** and equipped with a modified monoidal product, define nategories which satisfy additional regularity properties. This formalizes negative information categorically in a way that makes negative and positive morphisms equal citizens.

Diagrammatic Negative Information

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The flow of information through a complex system can be readily understood with category theory. However, negative information (e.g., what is *not* possible) does not have an immediately evident categorical representation. The formalization of nategories using unconventional composition addresses this issue, and lets imposed limitations on categories be considered. However, traditional nategories abandon core categorical constructs and rely on extensive mathematical development. This creates a divide between the consideration of positive and negative information composition. In this work, we show that negative information can be considered in a natural categorical manner. This is aided by functor string diagrams, a novel flexible diagrammatic approach that can intuitively show the operation of hom-functors and natural transformations in expressions. This insight reveals how to consider the composition of negative information with foundational categorical constructs without relying on enrichment. We present diagrammatic means to consider not only nategories, but preorders more broadly. This paper introduces diagrammatic methods for the consideration of triangle inequalities and co-designs $\mathbf{DP}/\mathbf{Feas}_{\mathbf{Bool}}$, showing how important cases of negative information composition can be categorically and diagrammatically approached. In particular, we develop systematic tools to rigorously consider imposed limitations on systems, advancing our mathematical understanding, and present intuitive diagrams which motivate widespread adoption and usage for various applications.

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Richer notions of **functionalities** and **resources**:

Spatial-temporal resources

Poly?

Linear Logic

Modeling Choice in Co-Design

Marius Furter

July 20, 2021

Abstract

This report describes a method for modeling free and forced choice within Co-Design. In a free choice among a set, one has control over which option is selected, while in a forced choice one does not. Given a preorder $\mathcal{P}$ describing resources or functionalities, a free choice among a subset of $\mathcal{P}$ acts like a meet. Dually, a forced choice acts like a join. Moreover, the two types of choice distribute over one another. Based on this, we construct a universal model for choice on a preorder using the free completely distributive lattice $UL\mathcal{P}$. Feasibility relations are then extended to these models. Along the way, we illustrate how to work within $UL\mathcal{P}$ and provide results that simplify calculations. The definitions presented here have been implemented in Haskell.

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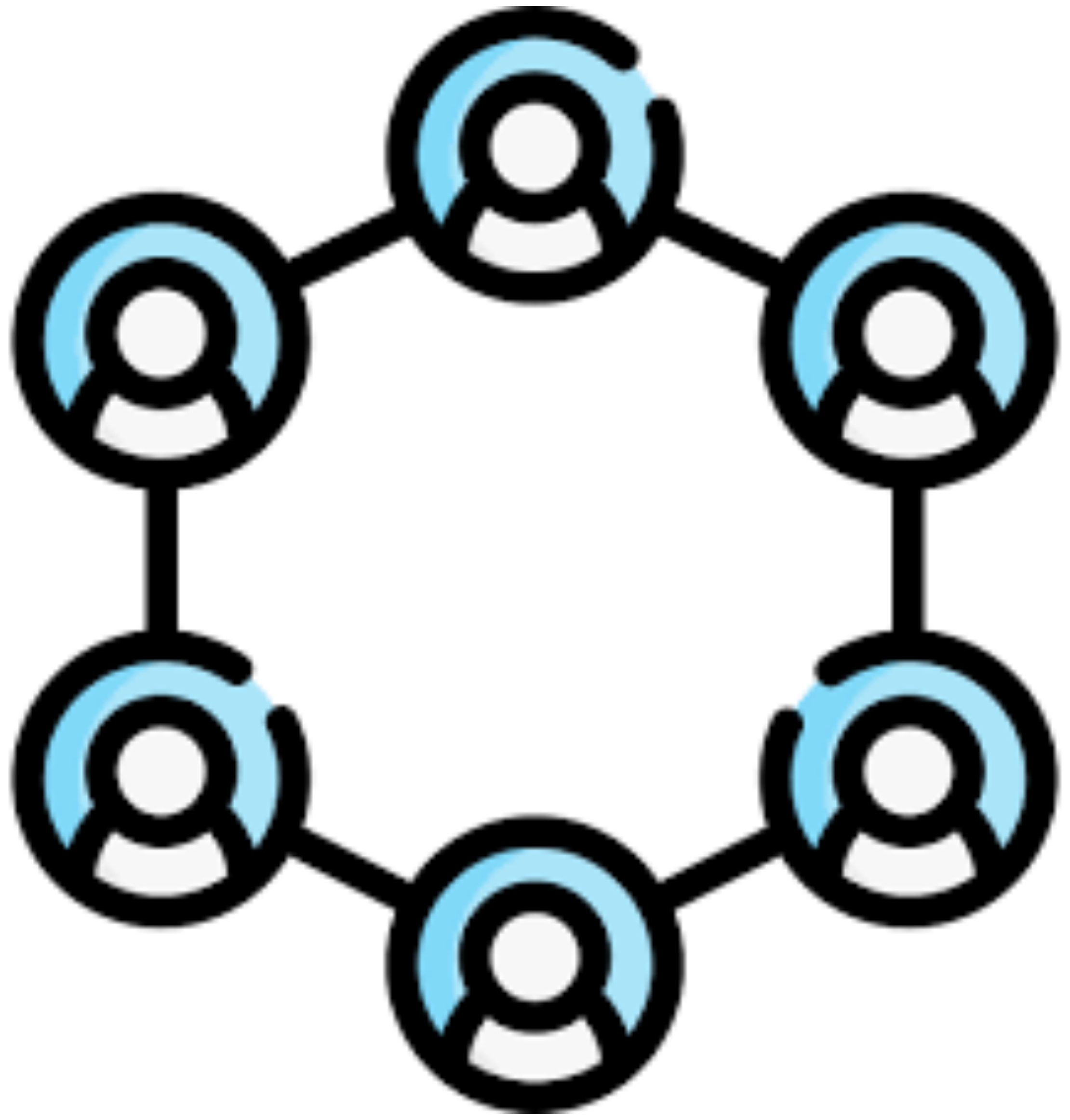
Coming up with the diagrams

Humans in the loop

Interfaces



Games

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Uncertainty

… in the **interconnection** of diagrams … (diagrams can be functionality/resources)

… in **feasibility** relations …

… in the **functional decomposition** …

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Societal Applications

Mobility, networks, infrastructure
Strategic interactions at all levels

Mission-driven autonomy

Aerospace, automotive, production chains, energy and data networks

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User-friendly Tools

Collaborative, intellectually tractable

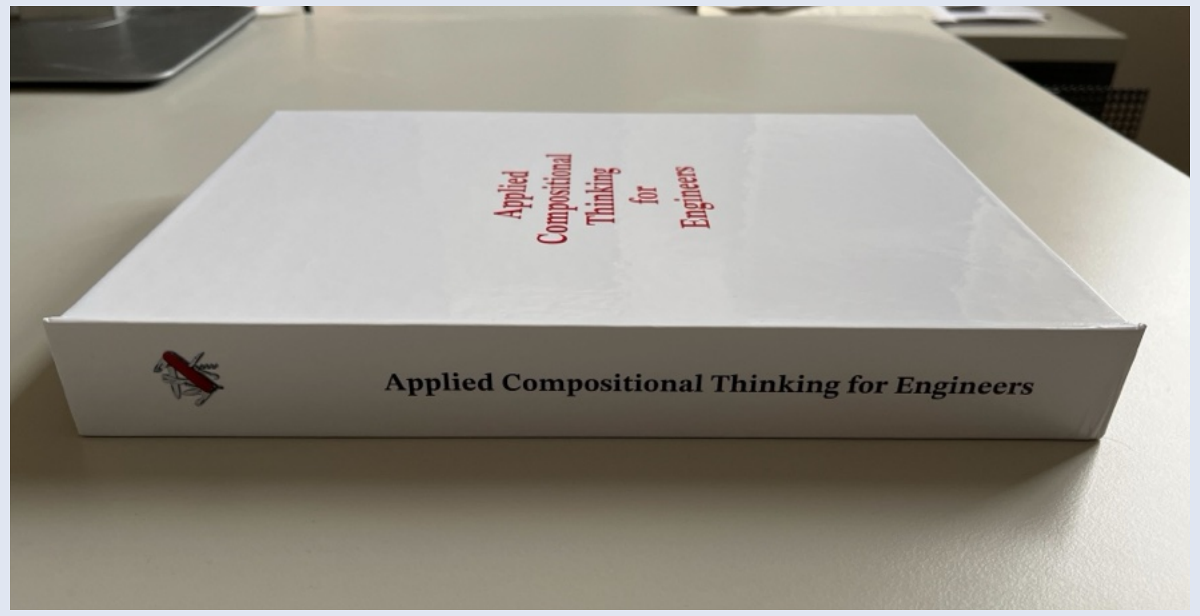
Authorities & Industry

Literature, workshops, classes

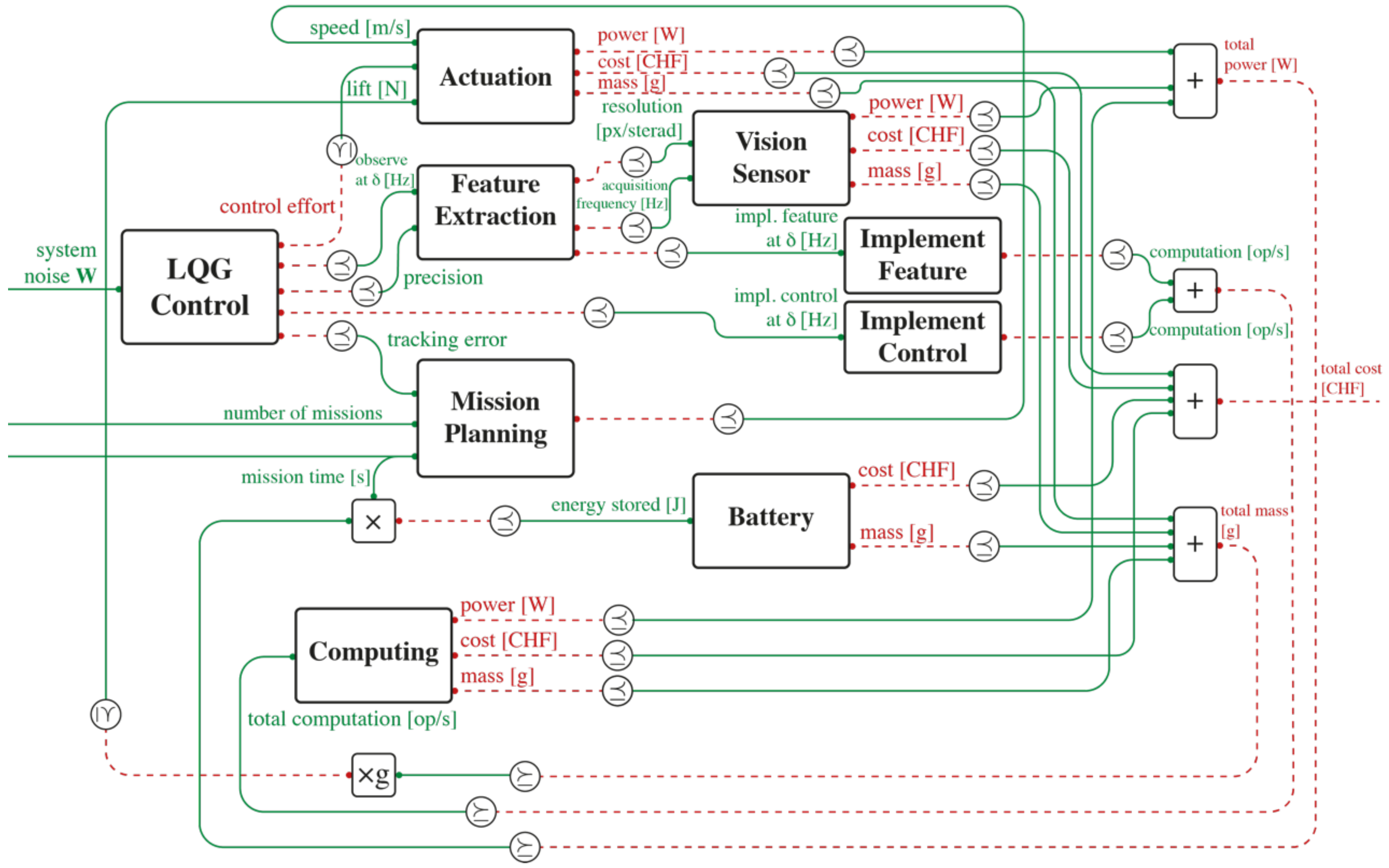
Take-aways

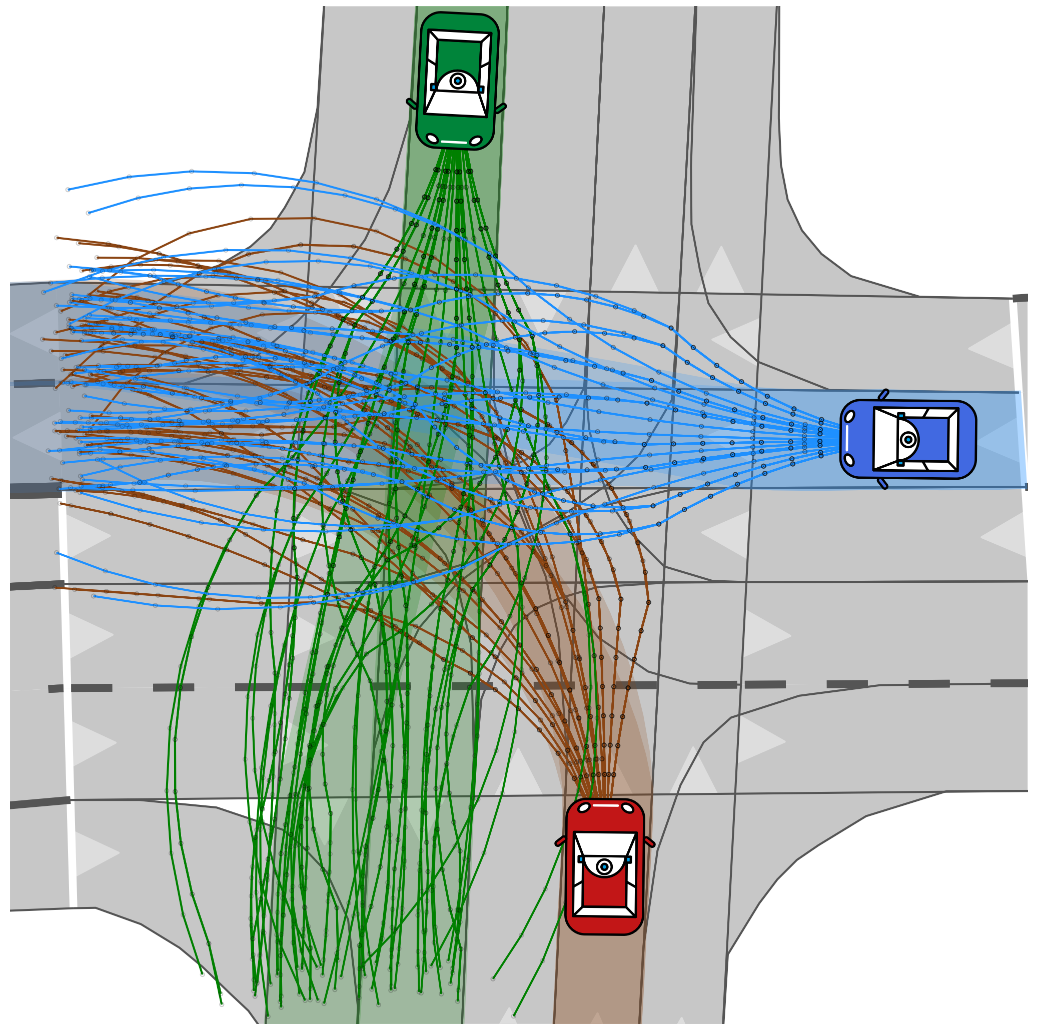
- A new approach to **co-design** designed to work **across fields** and **scales**.
- It is:
  - **Compositional** horizontally and hierarchically.
  - Supports both **data-driven** and **model-based** components.
  - **Computationally tractable**.
  - **Intellectually tractable.**
- Future: extend **modeling** and **algorithmic** capabilities

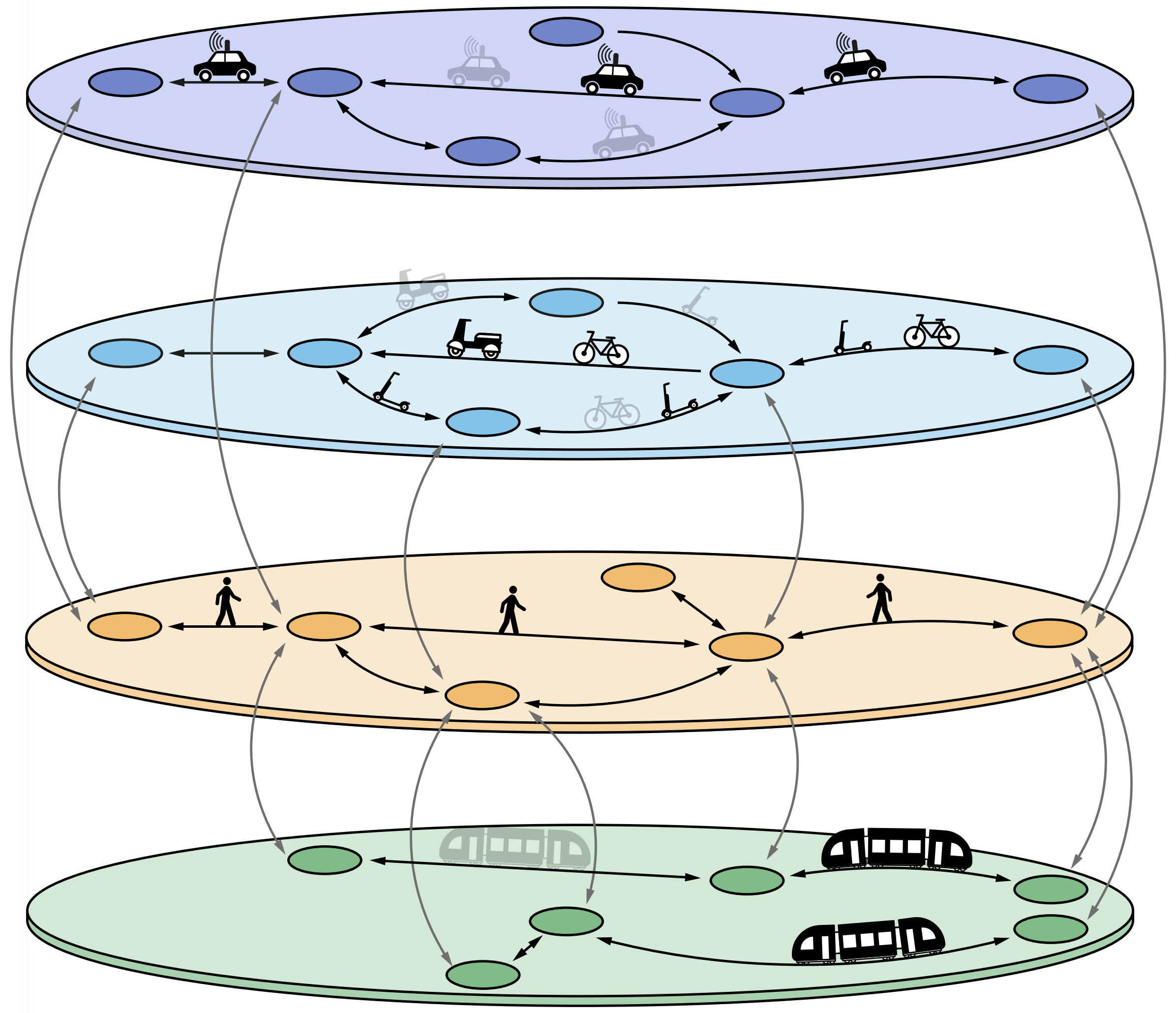
- We need to account for **strategic interactions** of **designers**:
  - **Posetal games**: A new class of games, where **utilities** are **posets**
- Future: **uncertainty** and **computational** schemes



Access the book at:
https://bit.ly/3qQNrdR







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Co-Design of autonomy, mobility

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Strategic Interactions

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- **Collaborators** for the presented works

Zelio Suter	**Dr. Jonathan Lorand**
Laura Guerrini	Dr. Saverio Bolognani
Dejan Milojevic	**Dr. Andrea Censi**
Nicolas Lanzetti	Prof. Florian Dörfler
Alessandro Zanardi	Prof. Marco Pavone
	Prof. Emilio Frazzoli

ETH zürich

Stanford University

Questions?

I'm hiring/welcoming visitors

zardini.mit.edu