Poly is an unreasonably effective abstraction; Might it relate to healthy systems?

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TOPOS INSTITUTE

Finding the Right Abstractions for Healthy Systems
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Outline

1. **Introduction**
   - Why am I here?
   - Sense-making
   - Accounting systems
   - Interfaces and interaction
   - Plan for the talk
2. Introduction to Poly
3. Unreasonable effectiveness of Poly
4. What is health?
5. Conclusion

Why am I here?

For reasons I don't fully understand, math is powerful.

- Math is at the core of science and technology.
- In whatever sense math *tracks* reality, it empowers *prediction*.
- We can say earlier something about what *will* happen later.
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- E.g. should we distinguish between health, wealth, and wisdom?

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The etymology of *health* points to a root of *wholeness*.

- This points us to the health of collectives rather than individuals.
- I am constituted by a collective, and I help constitute collectives.
- My health is measured by the extent to which both are *whole*.
- The question of wholeness is that of *integrity*: does it hold together?

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I'm here to think with you about math for helping systems be healthier.

Sense-making

What enables a collective to act in concert?

- What bring coherence, cooperation, coordination, integrity?
- What prevents friction, loss, stepping on toes, thwarting ourselves?
- Different parts of the collective see from different perspectives.
- Each acts according to their own mandate. What brings coordination?

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I think the word *sense* is appropriate.

- By *sense* I don't mean raw perception.
- Think Spidey-sense, sense of direction, of danger, of humor.
- Very complex situations are boiled down into something actionable.
- We talk about *making* sense; is this a pun, or can sense be made?

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Consider a snapshot of two math students, both wanting to succeed:

- Student A is faithfully copying down what the teacher says.
- Student B argues, points, gestures, explains, and then... gets it!
- The collective of sense-makers in B found a way to align.
- Having "made sense", student B performs better.

Working language

As a collective, we want to get a better sense of what health is.

- We may need to argue about it, work with it, gesture at it.
- In the end, we want to align our senses of health.
- Why does this entail “finding the right abstractions”?

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Language *works* in the sense of basic physics.

- Language like "pass the salt" can direct displacement of objects.
- DNA as a 4-letter language coding for amino acids also *works*.
- Computer programming languages direct changes in voltage levels.
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Language is$^?$ fast-moving pattern that can be unpacked in local context.

- I send you patterned air (pressure waves) about wanting salt.
- Coordinated signals (patterned neural) are sent around your brain.
- Each makes sense in the local context and the results cohere.
- Disputes between components can be resolved through language.

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I would guess that any healthy collective relies on appropriate language.

Mathematical fields as accounting systems

What does it mean that components resolve disputes through language?

- Each body part, employee in an org, member of a workshop...
- ...has its own view of the situation and set of commitments to honor.
- Accountability means being able to explain actions in collective terms.
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I think of mathematical fields as crystalized accounting systems.

- Arithmetic accounts for the flow of quantities, as in finance.
- Hilbert spaces account for the states of elementary particles, as in QM.
- Probability distributions account for likelihoods, as in game theory.
- Each is crystalized: hardened and coherent. Think laws of arithmetic.

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So math might help heal a system by improving communication within it.

Interfaces and interaction

We want math for improving communication within collectives.

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- Interfaces—aka surfaces, boundaries, membranes—are screens.
- Inside or outside the interface may be a whole collective.
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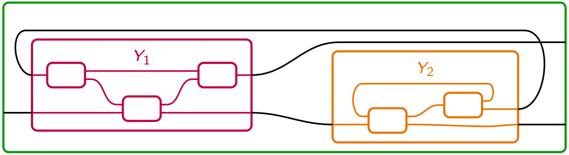
- The interface is just telling us the "input-output language".



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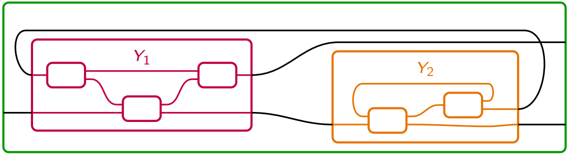

I want to present the category **Poly** and make the case for its use.

- I claim it's an unreasonably effective language for computer science.
- The main object of study can be understood as an interface.

Plan for the talk

During the remainder of the talk, I will:

- Give an intuitive mathematical introduction to **Poly**,
- Explain why I think it's unreasonably effective in computer science,
- Consider whether/how it help with our subject matter (health), and
- Conclude with a summary.

Outline

1 Introduction

2 **Introduction to Poly**
- Definition and intuition
- Lenses, Moore machines, and Mealy machines

3 Unreasonable effectiveness of Poly

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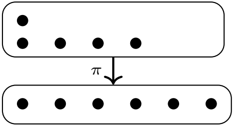
Definition and intuition

A *polynomial* p is essentially a data structure. Here are three viewpoints:

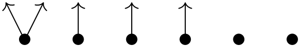
Algebraic

$$y^2 + 3y + 2$$

Bundle



Corolla forest



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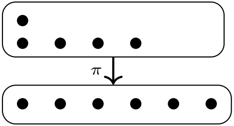

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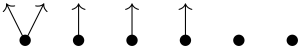

Cat. description: **Poly** = "sums of representables functors **Set** → **Set**".

- For any set S, let $y^S := \mathbf{Set}(S, -)$, the functor *represented* by S.
- Def: a polynomial is a sum $p = \sum_{i \in I} y^{P_i}$ of representable functors.
- Def: a morphism of polynomials is a natural transformation.
- Note that $I = p(1)$; this is a convenient fact. Write $p[i]$ for P_i.
- (We can use many other categories in place of **Set**, but let's not.)

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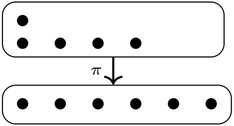

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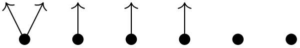

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Other ways to see a polynomial $p = \sum_{i \in I} y^{p[i]}$ as an interface:

- A set I of *types*; each type $i : I$ has a set $p[i]$ of *terms*.
- A set I of *problems*; each problem $i : I$ has a set $p[i]$ of *solutions*.
- A set I of *body positions*; each pos'n $i : I$ has a set $p[i]$ of *sensations*

Operations: $+, \times, \otimes, \triangleleft, [-,-], \left[\frac{-}{-}\right]$

Given two interfaces p, q, there are many ways to get another interface.

- For each we'll say the problems and solutions for resulting interface.
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Letting $p \coloneqq \sum_{i \in p(1)} y^{p_i}$ and $q \coloneqq \sum_{j \in q(1)} y^{q_j}$

$$p \times q = \sum_{(i,j)} y^{p[i]+q[j]} \qquad p \otimes q = \sum_{(i,j)} y^{p[i] \times q[j]}$$

$$p \triangleleft q = \sum_{i \in p(1)} \sum_{j \colon p[i] \to q(1)} y^{\sum_{x \in p[i]} q[jx]} \qquad [p,q] = \sum_{\varphi \colon p \to q} y^{\sum_{i \in p(1)} q[\varphi_1 i]}$$

Comonoids are categories

Poly has a lot of amazing surprises, as we'll see. Here's one.

- The substitution product $p \triangleleft q$ means plug q into p.
- So $y^2 \triangleleft (y+1) \cong y^2 + 2y + 1$. Not symmetric! $(y+1) \triangleleft y^2 = y^2 + 1$.
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In any mon'l cat'y, it's interesting to consider the monoids and comonoids.

- In the case of $(\mathbf{Poly}, y, \triangleleft)$, the comonoids are exactly categories!
- If $\mathcal{C}$ is a category, for any $c \in \mathrm{Ob}(\mathcal{C})$ define $\mathcal{C}[c] := \sum_{c' \in \mathrm{Ob}(\mathcal{C})} \mathcal{C}(c, c')$.

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- Then the associated polynomial is $p_{\mathcal{C}} := \sum_{c \in \mathrm{Ob}(\mathcal{C})} y^{\mathcal{C}[c]}$.
- Identities, codomains, and compositions are given by coherent maps

$$\epsilon \colon p_c \to y \qquad \text{and} \qquad \delta \colon p_c \to p_c \triangleleft p_c$$

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All that to say that comonoids in **Poly** are exactly categories!

- Maps between comonoids are not functors; they're "cofunctors".
- Ex: every Sy^S has a comonoid structure. Lawful lenses = cofunctors.
- Denote the category of categories and cofunctors by $\mathbf{Cat}^\sharp$.

Lenses, Moore machines, and Mealy machines

For any p, q as above, we have $\begin{bmatrix} q \\ p \end{bmatrix} = \sum_{i \in p(1)} y^{q(p[i])}$.

- In particular, we can regard $A, B \in \mathbf{Set}$ as constant polynomials.
- Then $\begin{bmatrix} A \\ B \end{bmatrix} = By^A$. Maps between these are "lenses".
- A map $\begin{bmatrix} A \\ B \end{bmatrix} \to \begin{bmatrix} A' \\ B' \end{bmatrix}$ is a natural transf'n $By^A \to B'y^{A'}$. It consists of
  - get: $B \to B'$
  - put: $B \times A' \to A$
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What's the point? The main math definition stuff is done. Let's get to it.

- A map $\begin{bmatrix} S \\ S \end{bmatrix} \to \begin{bmatrix} A \\ B \end{bmatrix}$ is a *Moore machine*. It consists of:
- State set S, a readout $f^{\text{rdt}} \colon S \to B$, and dynamics $f^{\text{dyn}} \colon S \times A \to S$.

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- A map $\begin{bmatrix} S \\ S \end{bmatrix} \to \begin{bmatrix} A \\ B \end{bmatrix}$ is a *Moore machine*. It consists of:
- State set S, a readout $f^{\text{rdt}} \colon S \to B$, and dynamics $f^{\text{dyn}} \colon S \times A \to S$.
- Given some initial $s_0 \in S$ and an input list $a_0, \ldots, a_n$, let...
- ...$s_{i+1} \coloneqq f^{\text{dyn}}(s_i, a_i)$ and $b_i \coloneqq f^{\text{rdt}}(s_i)$. Get output list $b_0, \ldots, b_n$.

Lenses, Moore machines, and Mealy machines

For any p, q as above, we have $\begin{bmatrix} q \\ p \end{bmatrix} = \sum_{i \in p(1)} y^{q(p[i])}$.

- In particular, we can regard $A, B \in \mathbf{Set}$ as constant polynomials.
- Then $\begin{bmatrix} A \\ B \end{bmatrix} = By^A$. Maps between these are "lenses".
- A map $\begin{bmatrix} A \\ B \end{bmatrix} \to \begin{bmatrix} A' \\ B' \end{bmatrix}$ is a natural transf'n $By^A \to B'y^{A'}$. It consists of
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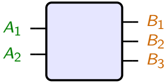
- A map $\begin{bmatrix} S \\ S \end{bmatrix} \to [Ay, By]$ is a *Mealy machine.*
- It consists of state set S and a function $S \times A \to S \times B$.
- Again, it can transform a list of inputs into a list of outputs.

Depicting Moore machine interfaces

Here's how we depict interfaces (A, B) for Moore machines:

If, e.g. $A = A_1 \times A_2$ and $B = B_1 \times B_2 \times B_3$, we will instead draw:



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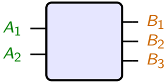
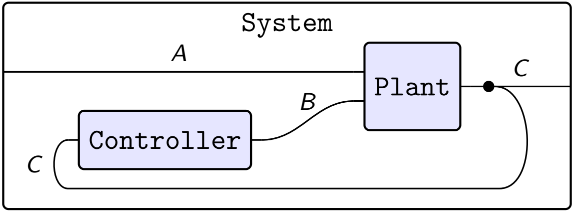


In **Poly** these two interfaces are denoted By^A and $B_1 B_2 B_3 y^{A_1 A_2}$.

Wiring diagrams

Here's a picture of a wiring diagram:



It includes three interfaces: `Controller`, `Plant`, and `System`.

$$\texttt{Controller} = By^{C} \qquad \texttt{Plant} = Cy^{AB} \qquad \texttt{System} = Cy^{A}$$

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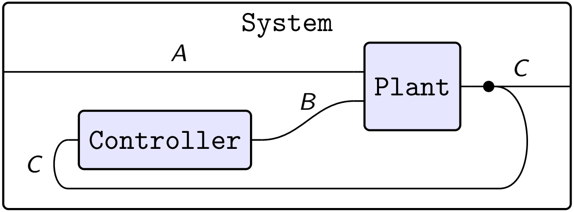


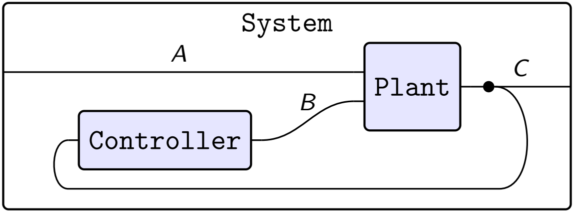
It includes three interfaces: Controller, Plant, and System.

$$\texttt{Controller} = By^{C} \qquad \texttt{Plant} = Cy^{AB} \qquad \texttt{System} = Cy^{A}$$

The wiring diagram represents a lens $\texttt{Controller} \otimes \texttt{Plant} \to \texttt{System}$.

$$By^{C} \otimes Cy^{AB} \longrightarrow Cy^{A}$$

Moore machines and wiring diagrams as lenses



To summarize what we've said so far:

- A wiring diagram (WD) is a lens, e.g. $By^C \otimes Cy^{AB} \longrightarrow Cy^A$.
- Each Moore machine is a lens, e.g. $Sy^S \to By^C$ and $Ty^T \to Cy^{AB}$.

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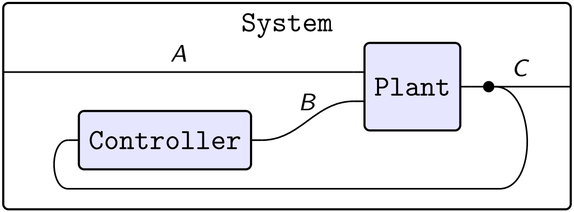

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- Each Moore machine is a lens, e.g. $Sy^S \to By^C$ and $Ty^T \to Cy^{AB}$.

We can tensor the Moore machines and compose to obtain $STy^{ST} \to Cy^A$.

- So a wiring diagram is a formula for combining Moore machines.
- The whole story is lenses (monomials), through and through.
- For "mode dependence" where interfaces can change, use gen'l polys.

Outline

1. Introduction

2. Introduction to Poly

3. **Unreasonable effectiveness of Poly**
   - Category theory in Computer Science
   - Functional programming
   - Databases and data migration
   - Dependent type theory
   - Effects handlers
   - Dynamic organizational systems
   - Synthetic programming
   - The “scientist category”
   - Mathematical properties of **Poly**

Category theory in Computer Science

Category theory has been useful in computer science.

- Functional programming languages, e.g. Haskell.
- Here, types are objects, programs are morphisms.
- The result is a cartesian closed category: tupling and function types.
- Side effects are handled by monads.

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Poly can add a lot to this story.

- First, note that it's already involved in many ways.
  - Algebraic data types are free monads on polynomial functors.
  - Initial algebras and final coalgebras for poly's are very common.
  - Lenses are maps between monomials.
- But we will see that **Poly** goes far beyond functional programming.
- We've seen it's relevant for finite state (Moore) machines. Also:
  - Databases and data migration,
  - Dependent type theory,
  - Effects handling,
  - Rewriting workflows,
  - Deep learning

Functional programming

In functional languages such as Haskell, you often see things like this:

```
data Foo y = Bar y y y | Baz y y | Qux | Quux
data Maybe y = Just y | Nothing
```

- These are polynomials: $y^3 + y^2 + 2$ and $y + 1$ respectively.
- They're "polymorphic" in that
  - they act on any Haskell type `Y` in place of the variable `y`, and
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- This the *algebraic data type* corresponding to $p_A \coloneqq 1 + A\mathcal{y}$.
- Every polynomial has an initial algebra and final coalgebra.
- The initial algebra of p_A is carried by $\sum_{n \in \mathbb{N}} A^n$, classic lists.
- The terminal coalgebra of p_A is carried by $A^{\mathbb{N}} + \sum_{n \in \mathbb{N}} A^n$, streams.

Databases and data migration

Databases are used throughout computer science.

- A database consists of a *schema*, the things and how they relate,...
- ...and *data*, which are examples of the things and their relationships.
- A useful CT story for this: schema $=$ category, data $=$ functor to $\mathbf{Set}$.
- Data migration means moving data from one schema to another.
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Often databases are considered ugly, but the math here is cat'ly very clean.

Dependent type theory

Dependent types are what proof assistants like Coq&Lean are based on.

- Idea: a type can depend on values of another type.
- Eg: a category consists of a type O of objects and then...
- ...for every $o_1, o_2 : O$, a type $M(o_1, o_2)$ of morphisms and then...
- ...identities, compositions, rules, all depending on the previous stuff.

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So the high-level language of proof assistants has semantics in **Poly**.

Effects handlers

In imperative programming, effects aren't unusual; they're the norm.

- The FP approach is to treat effects as scary, and use monads.
- But the result may be a pretty hairy mess of monads.
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Owen Lynch and I designed a language for effects handling using **Poly**.

- Say the REPL has interface $q \coloneqq \mathtt{String}\, y + y^{\mathtt{String}} + y$
- Say the "CPU" has interface $p \coloneqq \sum_{r,r':\mathtt{Reg}} \{\mathtt{ADD}, \mathtt{MUL}, \mathtt{OUT}\}y + \mathtt{BEQ}y^2$
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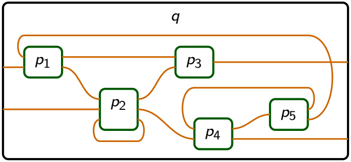
These are cat'ly nice: they form a distributive-monoidal double category.

Dynamic organizational systems

Earlier we discussed wiring diagrams interconnecting dynamical systems.



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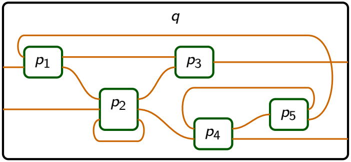

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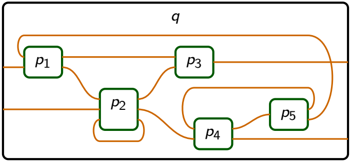

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In certain cases this can be done coherently: *dynamic operads*.

- Deep learning, prediction markets, Hebbian learning, open games.
- The way subordinates aggregate info changes based on what flows.

Synthetic programming

In synthetic programming, you have a type and you want a term of it.

- For example, you want a term of type $A \to (B \times C)$.
- The interface for a synthetic programmer can be given by $p : \mathbf{Poly}$.
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The workflow is to farm out problems and aggregate solutions.

- A map $\varphi \colon p \to q_1 \otimes \cdots \otimes q_N$ is a two-step process:
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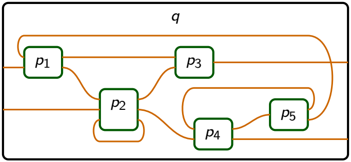
  - ...it returns a solution $\varphi_i^\sharp(x_1, \ldots, x_n) \in p[i]$.
- A map $\psi \colon p \to q_1 \triangleleft \cdots \triangleleft q_N$ is an $N + 1$-step process.
  - It sends each problem $i \in p(1)$ to a problem $j_1 \in q_1(1)$.
  - Given any solution $x_1 \in q_1[j_1]$, it returns a problem $j_2 \in q_2(1)$....
  - ...given any solution $x_N \in q_N[j_N]$, it returns a solution in $p[i]$.
- You get more examples by mixing and matching $\otimes, \triangleleft$.

The "scientist category"

Now instead, imagine *walking through* a diagram like this:



- You're in some state as you walk into q through a door on the left.
- You're shuttled to some p-box, which you then enter.
    - You find the room in some state, you interact with it for a while.
    - You either never leave, or you leave out a door on the right.
    - When you leave, the room is in a new state, and so are you.
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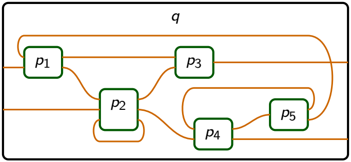

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This is operadic, a different semantic than the simultaneous dyn. sys's.

- It has applications as a VDSL for rewriting protocols.
- It has a **Poly** description: the category $\mathbf{Set}_*$ is enriched in $\mathbf{Cat}^\sharp$.

Mathematical properties of Poly

Finally, the category **Poly** is incredibly rich and well-behaved:

Mathematical properties of Poly

Finally, the category **Poly** is incredibly rich and well-behaved:

- Coproducts and products that agree with usual polynomial arithmetic;
- All limits and colimits;
- At least three orthogonal factorization systems;
- A symmetric monoidal structure $\otimes$ distributing over $+$;
- A cartesian closure q^p and monoidal closure $[p, q]$ for $\otimes$;
- Another nonsymmetric monoidal structure $\triangleleft$ that's duoidal with $\otimes$;
- A left $\triangleleft$-coclosure $\left[\begin{smallmatrix} - \\ - \end{smallmatrix}\right]$, meaning $\mathbf{Poly}(p, q \triangleleft r) \cong \mathbf{Poly}(\left[\begin{smallmatrix} r \\ p \end{smallmatrix}\right], q)$;
- An indexed right $\triangleleft$-coclosure (Myers?), i.e. $\mathbf{Poly}(p, q \triangleleft r) \cong \sum_{f\colon\, p(1) \to q(1)} \mathbf{Poly}(p \overset{f}{\frown} q, r)$;
- An indexed right $\otimes$-coclosure (Niu?), i.e. $\mathbf{Poly}(p, q \otimes r) \cong \sum_{f\colon\, p(1) \to q(1)} \mathbf{Poly}(p \overset{f}{\nearrow} q, r)$;
- At least eight monoidal structures in total;
- $\triangleleft$-monoids generalize Σ-free operads;
- $\triangleleft$-comonoids are exactly categories; bicomodules are data migrations. This is $\mathbb{C}\mathbf{at}^\sharp$.

See "A reference for categ'ical structures on **Poly**", arXiv: 2202.00534

Summary

The **Poly** ecosystem incredibly rich. It has ready-made abstractions for:

- Moore machines, Mealy machines, wiring diagrams,
- Dependent type theory,
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It's a great abstraction, but how does it help with understanding health?

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4. **What is health?**
    - Meeting the moment
    - Accountability
    - The health of this workshop: how to cohere?
5. Conclusion

Meeting the moment

It's unclear to me what health is or how to formally think about it.

- As said earlier, health means wholeness: a collective acts coherently.
- But then is a rock a degenerate case, or not of the right type?
- Should we distinguish between health, wealth, and wisdom?
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A healthy system is one that can *meet the moment*.

- When something threatens its cohesion, its integrity, it must meet it.
- It sees problems in advance, and it starts working to avoid them.

Accountability

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In each case, language is developed to translate between part and whole.

The health of this workshop: how to cohere?

We are here to understand how collectives coordinate to meet the moment.

- We're in the midst of massive and unsettling change on Earth.
- Humanity needs to meet the moment.
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Throughout this workshop, we can use our own coherence as a guide.

- Notice a failure to cohere? Look for the right way to think about why.
- What's at the core of this whole inquiry? What do we need to clarify?

Outline

1. Introduction
2. Introduction to Poly
3. Unreasonable effectiveness of Poly
4. What is health?
5. **Conclusion**
   - Summary

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- Our human collective works together by exchanging language.
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I’m looking forward to FRA with you; may we meet the moment!

Thanks! Comments and questions welcome...